

Due diligence regulation in informal auctions^{*}

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Abstract

A house purchase is for many people the largest financial decision of their lives. Yet 15–30% of American homebuyers waive inspections. Lawmakers have sought to institute a *no-waiver* policy preventing this practice. I propose a tractable model where buyers with heterogeneous valuations and risk aversion make offers to a risk-averse seller. Each offer (i) specifies a price and (ii) either includes an inspection contingency or waives inspections. The house’s repair cost is uncertain. It is shared between the winning buyer and the seller if the winning offer includes an inspection contingency.

I show that the prevalence of inspection waivers in a seller’s market can be explained by limited buyer bargaining power to negotiate seller remediation after an inspection. Even so, the no-waiver policy distorts risk-sharing between the seller and the winning buyer, thereby tending to decrease welfare. However, it reduces inequality, benefiting less wealthy (and thus more risk-averse) buyers by softening the competition they face from wealthier (and thus more risk-tolerant) buyers who would otherwise waive inspections. The policy can increase welfare by inducing homeowners to maintain their homes more efficiently before selling them.

Keywords: inspection waiver, due diligence, multidimensional auction, scoring auction, seller moral hazard, housing market, real estate

JEL classification codes: D44, D82, D86, K12, R30, R31, R38

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1 Introduction

Five million American homes change hands each year, for a total of roughly two trillion dollars (NAR, 2026a). For many people, the purchase of a house is the largest financial decision of their lives, and their house represents a large part of their wealth. Accordingly, prospective homebuyers often include an inspection contingency in their offers: If the offer is accepted, a professional will inspect the house, and based on the findings, the buyer can then renegotiate the terms of the sale or cancel it.

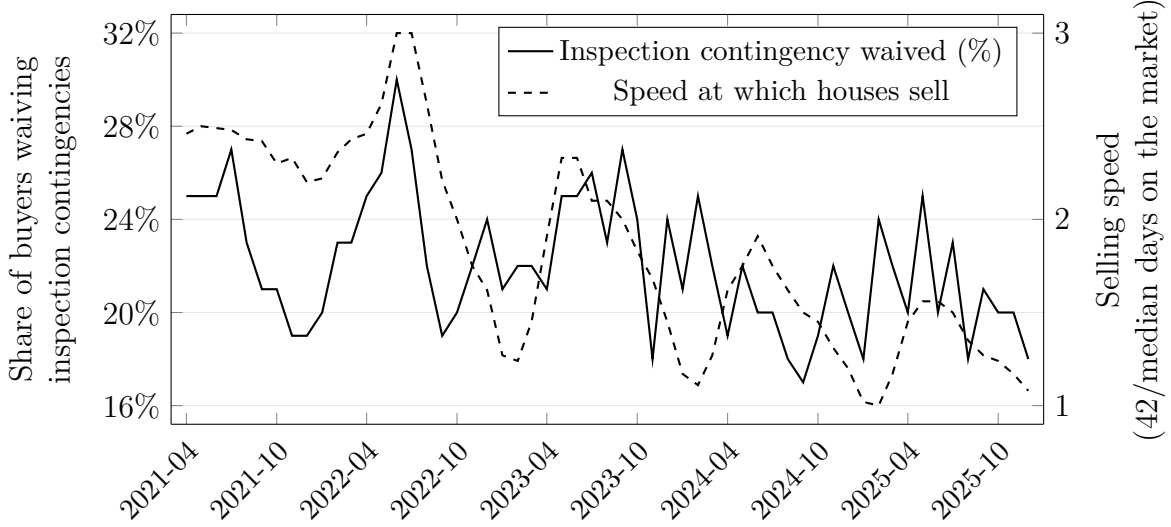
Inspections could serve at least three purposes. First, they allow buyers and sellers to share risk.¹ Of course, buyers who are better equipped to assume the risk of repair costs can waive inspections and gain a competitive advantage. For example, corporate buyers, flippers, developers may have an advantage over individual buyers, and wealthy individual buyers may have an advantage over low-income ones. Second, homeowners may not maintain their houses properly because they do not fully internalize the positive externalities of such maintenance on future buyers of the house. Inspections might mitigate this issue, inducing homeowners to maintain their houses more efficiently. Third, inspections may mitigate potential information asymmetries between buyers and the seller (Hayunga, 2018).²

Yet 15–30% of homebuyers waive inspections. A possible explanation is that in a “seller’s market,” where buyers have limited power to negotiate and houses sell fast, buyers waive inspections to make their offers more attractive (see Figure 1). Recent empirical evidence points in this direction (Mateen et al., 2026). The practice has attracted the attention of regulators, who are concerned that having limited bargaining power, buyers are forced to waive inspections. In Massachusetts, since October 16, 2025, sellers of residential real estate have been prohibited from (i) conditioning the acceptance of an offer on the prospective buyer’s agreement to waive inspections or (ii) accepting an offer if the buyer has informed the seller that he will waive inspections. A similar law is under

¹Coles and Smith (2026) estimate the surplus from a sale to be around 10% of the house price. Inspections reveal on average roughly \$11,000 worth of repairs (Repair Pricer, 2019). At the same time, for 62% of Americans, the largest emergency expense they could handle using only savings is less than \$5,000 (Federal Reserve Board, 2026). In addition to the five million existing homes, 600,000 new single-family houses are sold on average each year (U.S. Census Bureau and HUD, 2026).

²New York State Senate Bill S8729A states: “In competitive housing markets, some buyers waive inspection rights [...]. This practice transfers risk entirely to buyers [...].” Policymakers and no-waiver policy advocates have supported the policy on the grounds that it would level the playing field between buyers and sellers and among buyers (see Massachusetts Secretary of Housing and Livable Communities Ed Augustus’s statement (Massachusetts EOHLIC, 2025c), the New York Right to Home Inspection Initiative (NYSAHI, 2026), and Massachusetts state senator Will Brownsberger’s survey results and statements (Brownsberger, 2024)). The New York State Association of Realtors (Hettervig, 2025) has argued in the opposite direction. New York State Senate Bill S8729B (NY State Senate, 2026) states: “The practice of waiving inspections to win competitive offers has become common enough to be dangerous. [...] The seller, who knows the property, benefits. The buyer, who does not, absorbs all the risk.” Massachusetts state senator Michael O. Moore argues that “some sellers exploit this trend [i.e., buyers waiving inspections] when they are aware of critical flaws” (Moore, 2024).

Figure 1: The prevalence of waived inspection contingencies in a seller’s market



Note: the solid line denotes the share of homebuyers who waived inspection contingencies (Lautz, 2026); the dashed line denotes 42 divided by the median number of days on the market of realtors’ most recent sale (NAR, 2026b). The correlation between the two variables is 0.51.

consideration in New York.³

However, even if the buyers’ limited bargaining power in a seller’s market is behind inspection waivers, this does *not* imply that the practice is inherently harmful to buyers. A theoretical analysis is useful to (i) examine whether limited buyer bargaining power can explain the prevalence of inspection waivers in a seller’s market, (ii) gauge the welfare effects on buyers and sellers of a *no-waiver policy* that prevents buyers from waiving inspections, and (iii) study alternative forms of inspection regulation. These are the issues this paper studies.

In accordance with the different potential roles of inspections, I consider three settings. In the baseline setting, a risk-averse seller has an indivisible object for sale. Risk-averse buyers participate in an informal auction to buy the object. Each buyer has a two-dimensional private type consisting of his risk aversion parameter and his valuation for the object. Each buyer submits an offer that specifies two terms: (i) the price and (ii) whether an inspection contingency is attached or the buyer waives inspections. The seller chooses an offer to accept. The winning buyer pays the price to the seller and obtains the object. If the accepted offer includes an inspection contingency, an inspection also takes place. It reveals the object’s repair cost and effectively leads to a renegotiation of the price: An exogenous share of the repair cost is borne by the seller, and the remainder by

³See Massachusetts Regulation 760 CMR 74.00: Residential home inspection waivers (Massachusetts EOHL, 2025a) and New York State Senate Bill S8729B (NY State Senate, 2026). Canada has considered instituting “a legal right to a home inspection” (Department of Finance Canada, 2022). Québec requires real estate agents to advise buyers to include inspection contingencies in their offers, while both Québec and British Columbia have considered making inspections mandatory (BCFSA, 2022).

the buyer. This share captures institutional norms and the buyer’s bargaining power after an inspection. In a seller’s market, the winning buyer may have less power to negotiate seller remediation after an inspection. An inspection is costly to both the buyer and the seller due to, for example, inspection fees, mortgage interest payments, delays, paperwork, and time-on-the-market effects (Taylor, 1999). The buyers and the seller are *ex ante* symmetrically informed about the object’s uncertain repair cost. In the second setting, there is moral hazard. Before the buyers submit offers, the seller chooses whether to exert costly, unverifiable effort to maintain the object. Effort sufficiently decreases the expected repair cost later on and also (weakly) decreases the uncertainty about the repair cost, so that exerting effort is efficient. After effort has been chosen, the game proceeds as in the baseline setting. In the third setting, there is asymmetric information. The game is the same as in the baseline setting except that the seller now privately observes the repair cost before choosing an offer to accept.

Inspections and risk sharing. I first discuss the results from the baseline setting, where inspection contingencies allow buyers to share risk with the seller.

The prevalence of inspection waivers in a seller’s market. In the equilibrium of the baseline setting, the more risk-tolerant buyers (i.e., those with risk aversion below a certain threshold r^*) waive inspections, while the more risk-averse buyers include inspection contingencies in their offers. In exchange for the inspection contingency, they offer a higher price. Specifically, as the risk aversion parameter crosses r^* , the offered price jumps upward. I show that r^* —and therefore the share of non-contingent offers (i.e., offers that waive inspections)—is single-dipped in the buyers’ power to negotiate seller remediation after an inspection. When buyers have limited bargaining power, inspection contingencies transfer inefficiently little risk to the seller. A decrease in the buyers’ bargaining power exacerbates this inefficiency, leading even more buyers to waive inspection contingencies. On the other hand, when buyers have very high bargaining power, a decrease in that power causes the share of non-contingent offers to decrease. Thus, the results can explain the prevalence of non-contingent offers in a seller’s market, as seen in Figure 1. To the extent that buyers’ power to negotiate seller remediation is consistently low in the U.S., a larger share of buyers will waive inspections in times of especially low power to negotiate.

No-waiver policy welfare effects. However, the fact that limited bargaining power might incentivize buyers to waive inspections does not imply that a policy preventing them from doing so would be beneficial. The high frequency of non-contingent offers in a seller’s market could actually be beneficial to the extent that it is a response to the reduced effectiveness of inspections as a risk-sharing tool. In line with recent regulatory efforts, I consider a *no-waiver policy* that prevents buyers from committing not to perform

an inspection when they submit an offer. After a buyer's offer has been accepted, he is free to choose whether to perform an inspection. I show that in the baseline setting, the policy decreases total surplus and the seller's payoff by distorting risk-sharing. In the free market equilibrium (i.e., without the policy), buyers use inspections to share the risk with the seller, thereby making the most attractive offer at the lowest possible cost to them. The most risk-tolerant buyers waive inspections, bearing all the repair-cost risk themselves and avoiding inspection costs. On the other hand, the most risk-averse buyers do not waive inspections. The policy distorts risk-sharing. In the equilibrium under the policy, risk-tolerant buyers—who would otherwise waive inspections—perform inspections, thereby sharing with the seller risk that they are better equipped to bear themselves. As a result, the policy reduces the surplus generated when a risk-tolerant buyer wins the object. Therefore, the policy decreases total surplus realization by realization. The distortion in effective valuations also decreases the seller's payoff.

When buyers with higher valuations tend to have (weakly) lower risk aversion, the policy also reduces aggregate buyer surplus. The reason is that by inducing risk-tolerant buyers (who would otherwise waive inspections) to perform inspections, the policy decreases the rents they can extract thanks to their risk aversion heterogeneity. When valuations and risk aversion are negatively stochastically dependent, the risk-tolerant buyers are the ones who tend to win the object because they are both less risk-averse and willing to pay more. On the other hand, a *ban* of inspections raises aggregate buyer surplus.

Indeed, if wealth heterogeneity underlies buyers' types, it is plausible that wealthier buyers are less risk-averse and at the same time have a higher willingness to pay. Of course, an inequality-averse policymaker may then weight the risk-averse buyers' welfare more heavily. I show that if this is the case, she may find the no-waiver policy beneficial. The policy benefits the more risk-averse buyers by decreasing the competition they face from risk-tolerant buyers—whose choices to waive inspections are distorted. Conversely, the most risk-tolerant buyers are harmed by the policy. For example, the policy favors less wealthy young parents over wealthier people without school-age children as they compete for a house in a desirable school district. Likewise, the policy favors individual homebuyers over corporate ones and real estate developers.

An alternative policy. To the extent that the policymaker does not have strong enough distributional preferences favoring less wealthy buyers over wealthier ones and the seller, the no-waiver policy is unattractive. I consider an alternative policy, which instead of restricting the contract space—by preventing the inspection decision from being contractually agreed on—*enriches* the contract space. Namely, it requires buyers not only to commit to whether an inspection will be performed but also to pre-specify how the repair cost assessed by the inspection will be shared with the seller. The policy leads to more efficient risk-sharing between the winning buyer and the seller, thereby increasing

total and seller surplus. In practice, such a policy would most likely require that the parties be able to agree on a trustworthy inspector.⁴

Inspections and maintenance. In the setting with moral hazard, I show that in the free market equilibrium, the seller maintains the object only if the efficiency gains from maintenance are large enough. However, for intermediate levels of maintenance efficiency, the no-waiver policy induces the seller to maintain the object, which she would not do in the free market equilibrium. By doing so, the policy increases total and seller surplus. However, it reduces aggregate buyer surplus. As in the baseline setting, by inducing risk-tolerant buyers to inspect, the policy decreases the rents they can extract thanks to their risk aversion heterogeneity. In addition, by inducing maintenance, the policy mitigates all buyers' uncertainty about the repair cost, which also decreases the rents they can extract thanks to their risk aversion heterogeneity. At the same time, the reduction in the expected repair cost is competed away and captured by the seller.

I show that in some cases, object maintenance by the seller and inspection contingencies in the buyers' offers are strategic complements. As a result, there can be multiple free market equilibria: An equilibrium where the seller maintains can coexist with an equilibrium where she does not. On the other hand, under the no-waiver policy, there is a unique equilibrium, in which the seller maintains. Therefore, from a robustness perspective, the policy is beneficial because it improves the worst-case total (and seller) surplus.

Inspections and adverse selection. In the setting with adverse selection, I characterize two kinds of free market equilibria: one in which every buyer waives inspections and one in which every buyer submits a contingent offer. Strikingly, I show that the former equilibrium exists if and only if the total inspection cost (borne by the buyer and the seller) is excessively high, that is, higher than every buyer's disutility from bearing all the repair-cost risk on his own.⁵ This is because the ability of buyers to include inspection contingencies in their offers does not solve the problem of adverse selection. By waiving inspections, a buyer experiences adverse selection as his offer tends to be accepted when the repair cost is high. Conversely, when he submits a contingent offer, he experiences favorable selection. Therefore, buyers have very strong incentives to include inspection contingencies in their offers. As a result, when buyers waive inspections in the free market equilibrium, the no-waiver policy decreases total and seller surplus by inducing buyers to inspect when inspection costs are excessively high compared to the repair-cost risk. As in the baseline setting, the policy harms the most risk-tolerant buyers while benefiting the

⁴Of course, this is also a consideration in the status quo, where the way costs will be shared is not pre-agreed upon.

⁵Conversely, the latter equilibrium exists for sufficiently low inspection costs.

most risk-averse ones, and its effect on aggregate buyer surplus is negative when buyers with higher valuations tend to have (weakly) lower risk aversion.

After a discussion of related literature, section 2 presents the model. Sections 3.1, 3.2, and 3.3 study the baseline setting, the setting with moral hazard, and the setting with adverse selection, respectively. Section 4 concludes. Proofs are in the Appendix.

Related literature. The paper contributes to the literature on information acquisition in auctions, which has primarily studied information acquisition before bidding (see, e.g., Milgrom, 1981; Lee, 1984; Hausch and Li, 1993; Persico, 2000; Shi, 2012; Kim and Koh, 2020, 2022; Bobkova, 2024; Pernoud and Gleyze, 2024). Instead, this paper studies inspection contingencies as a contractual term that allows buyers to acquire information after their offer has been accepted. Outside auctions, there are works that study the (design of) incentives to acquire information when signing a contract (see, e.g., Crémer and Khalil, 1992; Compte and Jehiel, 2007). I show that in the setting with moral hazard, maintenance by the seller—which increases the object’s value (by decreasing the expected repair cost) and decreases the uncertainty over it—has an ambiguous effect on the probability that the winning offer is contingent. When it increases that probability, there can be equilibrium multiplicity.

This paper is closest to Cusumano and Pieroth’s (2025) work on due diligence in common value auctions. Cusumano and Pieroth (2025) consider the problem of a risk-neutral seller who can commit to whether she will allow risk-neutral buyers to perform an inspection and, if so, control the timing of the inspection. In my setting, the seller has no such commitment power and will consider any offer, contingent or not. Moreover, both the buyers and the seller are risk-averse, which captures a potentially major driving force for inspections in real estate transactions. Cusumano and Pieroth (2025) show that in low-stakes environments (i.e., when uncertainty over the value of the object is low), it is optimal for the seller to forbid inspections, while when the stakes are high, it is optimal to allow them.⁶ In this paper, if the seller had the power to forbid inspections, she would not want to do so, as this would distort risk-sharing, causing her surplus to decrease. Conversely, the seller can benefit from preventing buyers from waiving inspections, as this would incentivize her to more efficiently maintain the object before the auction. Daley et al. (2024) also study due diligence, in a dynamic model where the winning bidder (i.e., the acquirer of a firm) starts conducting due diligence and chooses whether and when to complete the transaction. They show that although the winning bidder engages in excessive due diligence in equilibrium, allowing for due diligence can increase total and seller surplus.⁷

⁶Allowing buyers to inspect the object before bidding is never optimal.

⁷For the role of due diligence in mergers and acquisitions, see Gole and Hilger (2009). For empirical work on due diligence, see Marquardt and Zur (2015) and Wangerin (2019). For a survey of real estate auctions, see Quan (1994).

This paper also contributes to the literature on multidimensional auctions. Bidders both have two-dimensional types and submit two-dimensional offers specifying the price and whether an inspection contingency is attached. Unlike many models with multidimensional private information (Manelli and Vincent, 2006; Daskalakis et al., 2017), the model I propose is tractable. As in scoring auctions (Che, 1993; Asker and Cantillon, 2008), the seller’s preferences over bids can be summarized by a one-dimensional sufficient statistic: the *effective bid*, which incorporates the seller’s disutility from bearing part of the repair-cost risk. The seller always accepts the offer with the highest effective bid. A buyer can achieve any level of the effective bid with either a contingent or a non-contingent offer. His preferences between the two are independent of the level of the effective bid. As a result, the buyer’s problem of choosing whether to include an inspection contingency is separable. This simplification then allows us to also summarize a buyer’s preferences over effective bids by a one-dimensional sufficient statistic, his *effective valuation*, which encodes his two-dimensional type. The game then reduces to a first-price auction.

The paper also contributes to the literature on auctions with risk-averse bidders and/or sellers (Holt, 1980; Riley and Samuelson, 1981; Milgrom and Weber, 1982; Matthews, 1983; Maskin and Riley, 1984; Matthews, 1987; Smith and Levin, 1996; Esó and White, 2004; Esó, 2005; Hu et al., 2010; Li et al., 2015; Baisa, 2017; Klose and Schweinzer, 2022; Fioriti and Hernández-Chanto, 2022; Gershkov et al., 2022; Pease and Whitmeyer, 2026).⁸ A handful of papers allow for risk aversion heterogeneity across buyers (Cox et al., 1982; Hu et al., 2018a; Brisset et al., 2020; Fioriti and Hernández-Chanto, 2022), while others allow buyers to also differ in their valuations (Cox et al., 1988; Che and Gale, 2006; Gentry et al., 2015; Zeithammer and Stich, 2025; Carrasco and Hernández-Chanto, 2026). Hu et al. (2018b) study English auctions in which the seller is risk-averse and each buyer has a one-dimensional private type that can affect both his valuation and his risk aversion. To the best of my knowledge, this work comes closest to considering, as this paper does, a risk-averse seller and buyers who differ in both their valuation and their risk aversion.

Finally, the paper relates to work on real estate transactions (see, e.g., Mayer, 1995; Lusht, 1996; Quan, 2002; Harding et al., 2003; Merlo and Ortalo-Magné, 2004; Genesove and Han, 2012; Han and Strange, 2015; Chow et al., 2015; Albrecht et al., 2016; Hayunga and Munneke, 2021; Arefeva and Meng, 2021; Arefeva, 2025; Lin et al., 2025; Genesove and Hansen, 2025). While formal auctions are rare, informal auctions, or “bidding wars,” are common (Han and Strange, 2014; Choi et al., 2026; Mateen et al., 2026). Previous works have studied non-price competition in real estate, estimating the premiums paid (i) by mortgage-financed buyers relative to all-cash buyers (Han and Hong, 2024; Reher and Valkanov, 2024) and (ii) by buyers with subject-to-resale contingencies (Shilling et al., 1992).

⁸Vasserman and Watt (2021) review the literature on risk aversion in auctions.

2 The model

There are $N \geq 2$ buyers (he) and a seller (she) of a single indivisible object, such as a house. The buyers are indexed by $i \in \mathcal{N} := \{1, \dots, N\}$.

The interaction. The buyers participate in an informal auction. Each buyer submits an offer $(b_i, d_i) \in \mathbb{R}_+ \times \{0, 1\}$ to the seller, where b_i is the price and d_i indicates whether the offer is contingent. $d_i = 1$ indicates a contingent offer. The seller then picks one offer to accept.⁹ If the accepted offer (b_i, d_i) is contingent (i.e., $d_i = 1$), the object's repair cost c is revealed, the winning buyer pays b_i to the seller, obtains the object, and pays $(1 - \zeta)c$ to repair it, while the seller bears the *seller remediation* cost ζc , whether by making repairs herself or through concessions such as a repair credit or a price reduction. $\zeta \in (0, 1)$ captures the buyer's bargaining power after the inspection.¹⁰ If the accepted offer is non-contingent, the winning buyer pays b_i to the seller, obtains the object, and pays c to repair it.¹¹ In section 3.1, the buyers and the seller are symmetrically informed. In section 3.3, the seller privately observes c before accepting an offer. Section 3.2 considers the case where the seller can maintain the object before receiving offers. Each of these sections describes the timeline of the game in more detail.

Payoffs. Buyer i receives Bernoulli payoff $\pi_B(b_i, d_i; v_i, r_i | \mathcal{I}_i) := v_i - b_i - d_i \kappa_B - \mathbb{E}[(1 - \zeta d_i)c | \mathcal{I}_i] - r_i \text{Var}((1 - \zeta d_i)c | \mathcal{I}_i)$ from winning the object, where v_i is his valuation (i.e., his willingness to pay for the object absent risk), r_i his risk aversion, and $\mathcal{I}_i$ his information set, at which he knows his type, his offer, and that his offer has been accepted. He earns a payoff of 0 from losing. The seller receives Bernoulli payoff $\pi_S(b_i, d_i; r_S | \mathcal{I}_S) := b_i - d_i \kappa_S - \mathbb{E}[\zeta d_i c | \mathcal{I}_S] - r_S \text{Var}(\zeta d_i c | \mathcal{I}_S) = b_i - d_i \kappa_S - d_i \zeta \mathbb{E}[c | \mathcal{I}_S] - d_i r_S \zeta^2 \text{Var}(c | \mathcal{I}_S)$ from accepting buyer i 's offer, where $r_S \geq 0$ is the seller's risk aversion and $\mathcal{I}_S$ is her information set when she accepts the offer. The seller's valuation of the object is normalized to 0. $\kappa_B, \kappa_S \geq 0$ are the costs (e.g., inspection fees, delays, paperwork) incurred by the winning buyer and the seller, respectively, when an inspection is performed. Denote the total inspection cost by $\kappa := \kappa_B + \kappa_S$.¹²

Information. c and $(v_i, r_i)_{i=1}^N$ are independent. c has cumulative distribution function (CDF) F_c , density $f_c : [0, \bar{c}] \rightarrow \mathbb{R}_+$ positive on $(0, \bar{c})$, $\bar{c} > 0$, with mean $\mu := \int_0^{\bar{c}} c f_c(c) dc > 0$ and variance $\sigma^2 := \int_0^{\bar{c}} (c - \mu)^2 f_c(c) dc > 0$. $(v_i, r_i)_{i \in \mathcal{N}}$ are distributed independently and

⁹Assumption 1 guarantees that it is always optimal for the seller to accept one of the offers.

¹⁰Waiving inspections can be less risky for buyers if the seller has the house inspected before listing it and shares the report with prospective buyers.

¹¹For simplicity, a sales contract is assumed to always go through. In practice, roughly 5% of sales are terminated (Lautz, 2025). For an analysis of first-price auctions where the winner can withdraw after learning the object's value, see Asker (2000).

¹²In the U.S., the fee for a home inspection is typically \$300 to \$500 (Freddie Mac, 2025).

identically across buyers with CDF F_{vr} , continuous density $f_{vr} : [\underline{v}, \bar{v}] \times [\underline{r}, \bar{r}] \rightarrow \mathbb{R}_{++}$, $\bar{r} > \underline{r} \geq 0$, $\bar{v} > \underline{v} > 0$.¹³ Write $f_r(r) := \int_{\underline{v}}^{\bar{v}} f_{vr}(v, r) dv$, $f_{v|r}(v|r) := f_{vr}(v, r)/f_r(r)$, $F_{v|r}(v|r) := \int_{\underline{v}}^v f_{v|r}(t|r) dt$, and $F_r(r) := \int_{\underline{r}}^r f_r(t) dt$. $r_S \geq 0$ is common knowledge. (v_i, r_i) is buyer i 's private information.¹⁴

I maintain assumption 1, which guarantees that the expected surplus from trade is always positive, so that the seller always accepts an offer.

Assumption 1. $\underline{v} > \max\{\mu + \bar{r}\sigma^2, \kappa + \mu + (1 - \zeta)^2 \bar{r}\sigma^2 + \zeta^2 r_S \sigma^2, \kappa + (1 - \zeta)\mu + \zeta \bar{c} + (1 - \zeta)^2 \bar{r}\sigma^2\}$.

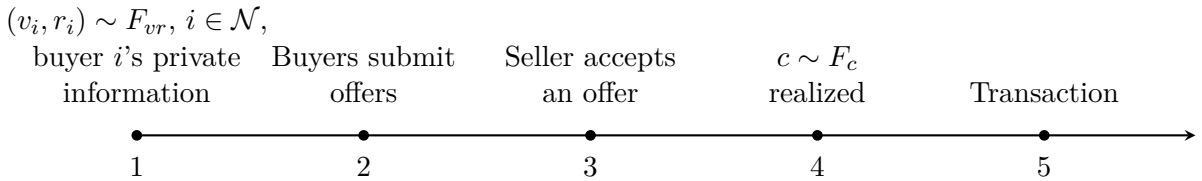
3 Analysis

Section 3.1 analyzes the equilibrium and policy effects in the baseline model of section 2. Section 3.1.4 studies the stated-share policy, which requires offers to state how the repair cost revealed by the inspection will be shared. Section 3.2 adds moral hazard to the model. It introduces a stage, before the object is put up for sale, in which the seller can exert unobservable costly effort to (efficiently) maintain the object (i.e., to decrease the repair cost c after the sale). Section 3.3 adds adverse selection to the model. It analyzes the case where the seller privately observes the state c of the object before accepting an offer.

3.1 The baseline model: inspections and risk sharing

Figure 2 presents the timeline of the baseline model. To simplify notation, we drop information sets from agents' payoffs.

Figure 2: Baseline game: timeline



¹³The results of the main text hold if F_c is only continuous (that is, atomless), with $F_c(t) > 0$ for every $t > 0$ and $F_c(t) < 1$ for every $t < \bar{c}$, and if f_{vr} is bounded rather than continuous. The results of sections 3.1 and 3.2 depend on the distribution of c only through its mean and variance.

¹⁴The possibility of resale could introduce a common value component into buyers' valuations. However, in many real estate markets, there is ample information about price trends and closing prices of several similar properties, so that the informational content of other buyers' offers is negligible. Therefore, the assumption of independent private values is indeed common in the literature on residential real estate (see e.g., Adams et al., 1992). Also, c is a common value component that captures the objective quality of the house.

3.1.1 The free market equilibrium

We first study the free market equilibrium. The seller's payoff from accepting offer (b_i, d_i) is $\pi_S(b_i, d_i; r_S) = \tilde{b}(b_i, d_i)$, where $\tilde{b}(b_i, d_i) := b_i - d_i\omega$ is the *effective bid* and $\omega := \kappa_S + \zeta\mu + \zeta^2\sigma^2 r_S$. In equilibrium, she accepts the highest effective bid. A buyer i who submits offer (b_i, d_i) and wins earns payoff $\pi_B(b_i, d_i; v_i, r_i) = V(d_i, v_i, r_i) - \tilde{b}(b_i, d_i)$, where

$$V(d_i, v_i, r_i) := \begin{cases} v_i - \mu - r_i\sigma^2 & \text{if } d_i = 0, \\ v_i - \kappa - \mu - r_i(1 - \zeta)^2\sigma^2 - \zeta^2\sigma^2 r_S & \text{if } d_i = 1. \end{cases}$$

Any given effective bid can be made in two ways: with or without a contingency (i.e., $d_i = 1$ or $d_i = 0$). Each buyer chooses between the two to maximize $V(d_i, v_i, r_i)$. Lemma 1 describes the equilibrium contingency choices.

Lemma 1. In any equilibrium where buyer i wins the object with positive probability (after observing (v_i, r_i)), if $r_i > r^*$ (resp. $r_i < r^*$), then $d_i = 1$ (resp. $d_i = 0$), where $r^* := \kappa/[\zeta(2 - \zeta)\sigma^2] + \zeta r_S/(2 - \zeta)$ is single-dipped in ζ .¹⁵

Denote buyer i 's *effective valuation* in the free market equilibrium by

$$V^*(v_i, r_i) := \max_{d_i \in \{0,1\}} V(d_i, v_i, r_i) = \begin{cases} V(0, v_i, r_i) & \text{if } r_i < r^*, \\ V(1, v_i, r_i) & \text{if } r_i \geq r^*. \end{cases}$$

Given the equilibrium contingency choices pinned down by Lemma 1, each buyer i 's two-dimensional problem of choosing a price b_i and a contingency term d_i reduces to choosing his effective bid $\tilde{b}_i$. Thanks to this simplification, the buyer's two-dimensional type (v_i, r_i) also reduces to a one-dimensional type $V^*(v_i, r_i)$. The game reduces to a first-price auction in which buyer i chooses his effective bid and his (effective) valuation is $V^*(v_i, r_i)$ with CDF $F_V^*(s) := \int_{\underline{r}}^{\bar{r}} \int_{\underline{v}}^{\bar{v}} \mathbf{I}\{V^*(v, r) \leq s\} f_{vr}(v, r) dv dr$. Thus, $F_V^*(s)^{N-1}$ is the probability that every other buyer's effective valuation is less than s . Notice that $V^*(v_i, r_i)$ is an equilibrium object, and thus so is F_V^* . A policy that changes the buyers' inspection choices will also change their effective valuations.

Proposition 1. There is a unique symmetric monotone equilibrium: Each buyer i submits offer $(b^*(v_i, r_i), d^*(r_i))$ with $d^*(r_i) = \mathbf{I}\{r_i \geq r^*\}$ and $b^*(v_i, r_i) = \tilde{b}^*(V^*(v_i, r_i)) + d^*(r_i)\omega$, where $\tilde{b}^*(s) := s - \int_{V^*(\underline{v}, \bar{r})}^s F_V^*(t)^{N-1} dt / F_V^*(s)^{N-1}$. The buyer with the highest $V^*(v_i, r_i)$ wins the object.¹⁶

¹⁵ r^* is decreasing in ζ where $\zeta^2 r_S \sigma^2 < (1 - \zeta)\kappa$ and increasing where $\zeta^2 r_S \sigma^2 > (1 - \zeta)\kappa$; both sets are non-empty if and only if $\kappa r_S > 0$.

¹⁶An equilibrium is monotone if the effective bid $\tilde{b}^*(s)$ is increasing in s . The equilibrium strategy is unique (in the class of symmetric monotone equilibria) for every (v_i, r_i) such that $r_i \neq r^*$ and $(v_i, r_i) \neq (\underline{v}, \bar{r})$. Throughout, we restrict attention to symmetric monotone equilibria.

Notice that there is an upward jump in the price $b^*(v_i, r_i)$ offered by buyer i when r_i crosses the threshold r^* . Denote by $M^* := \max_i V^*(v_i, r_i)$ the winning buyer's effective valuation. The seller's equilibrium realized surplus is $\tilde{b}^*(M^*)$. The winning buyer's equilibrium realized surplus is $M^* - \tilde{b}^*(M^*)$. M^* is the equilibrium realized total surplus. Buyer i 's interim expected surplus is $u^*(v_i, r_i) := \int_{V^*(\underline{v}, \bar{r})}^{V^*(v_i, r_i)} F_V^*(t)^{N-1} dt$.

Effect of buyers' bargaining power on inspection waiver prevalence. Lemma 1 and Proposition 1 show that r^* —and therefore the equilibrium share $F_r(r^*)$ of offers with waived inspections—is single-dipped in the buyers' post-inspection bargaining power ζ . When the buyers' post-inspection bargaining power is low, inspection contingencies transfer inefficiently little risk to the seller. A further decrease in the buyers' bargaining power ζ then exacerbates this inefficiency, thereby causing even more buyers to waive inspection contingencies (i.e., r^* increases). The opposite holds when buyers have inefficiently high bargaining power: In that case, a decrease in their bargaining power increases the prevalence of inspection contingencies.

Thus, the results can explain the prevalence of waived inspection contingencies in a seller's market, as seen in Figure 1. The evidence suggests that in the U.S. real estate market, the buyers' post-inspection bargaining power is consistently low, and in periods where it is especially low, a larger share of buyers waive inspections.

3.1.2 Equilibrium under the no-waiver policy

Now, consider a policy that prevents buyers from waiving inspections (i.e., committing not to do an inspection when they submit an offer), or, equivalently, prevents sellers from accepting offers with waived inspections. After a buyer's offer has been accepted, he is free to choose whether to have an inspection performed or not. This policy effectively limits the contract space: Offers include a price but no commitment as to whether there will be an inspection.

If buyer i 's offer is accepted, it is strictly optimal for him to do an inspection if and only if $r_i > r^n := (\kappa_B - \zeta\mu)/(\zeta(2 - \zeta)\sigma^2)$. For simplicity, we maintain the following assumption, which implies that in the no-waiver policy equilibrium, every buyer inspects.

Assumption 2. $\kappa_B \leq \zeta\mu + \zeta(2 - \zeta)\underline{r}\sigma^2$.

For the assumption to hold, it is sufficient that the inspection cost κ_B borne by the buyer is not higher than the share ζ of the average repair cost μ . Indeed, it is reasonable to expect that ex post, after their offer has been accepted, all buyers would want to do an inspection.

The seller's payoff from accepting offer b_i is $\pi_S(b_i, 1; r_S) = \tilde{b}(b_i, 1)$. In equilibrium, she accepts the highest bid. A buyer i who submits offer b_i and wins earns payoff

$\pi_B(b_i, 1; v_i, r_i) = V(1, v_i, r_i) - \tilde{b}(b_i, 1)$. Buyer i 's effective valuation is now $V(1, v_i, r_i)$, with CDF $F_V^p(s) := \int_{\underline{r}}^{\bar{r}} \int_{\underline{v}}^{\bar{v}} \mathbf{I}\{V(1, v, r) \leq s\} f_{vr}(v, r) dv dr$.

Proposition 2. Under the no-waiver policy, there is a unique symmetric monotone equilibrium: Each buyer i submits price $b^p(v_i, r_i) = \tilde{b}^p(V(1, v_i, r_i)) + \omega$ and, if he wins, he has an inspection performed, where $\tilde{b}^p(s) := s - \int_{V(1, \underline{v}, \bar{r})}^s F_V^p(t)^{N-1} dt / F_V^p(s)^{N-1}$. The buyer with the highest $V(1, v_i, r_i)$ wins the object.

Denote by $M^p := \max_i V(1, v_i, r_i)$ the winning buyer's effective valuation. The seller's equilibrium realized surplus is $\tilde{b}^p(M^p)$. The winning buyer's equilibrium realized surplus is $M^p - \tilde{b}^p(M^p)$. M^p is the equilibrium realized total surplus. Buyer i 's interim expected surplus is $u^p(v_i, r_i) := \int_{V(1, \underline{v}, \bar{r})}^{V(1, v_i, r_i)} F_V^p(t)^{N-1} dt$.

3.1.3 No-waiver policy effects

Assume in this section that $r^* > \underline{r}$, so that the no-waiver policy has an effect.

Effect on total and seller surplus. Proposition 3 shows that the policy causes an FOSD-decrease in total surplus and seller surplus.

Proposition 3. The policy FOSD-decreases

- (i) total surplus: For every realization $(v_i, r_i)_{i \in \mathcal{N}}$ of the buyers' types, $M^p \leq M^*$; and
- (ii) seller surplus: For every x , $\Pr(\tilde{b}^p(M^p) \leq x) \geq \Pr(\tilde{b}^*(M^*) \leq x)$.¹⁷

In making the most attractive offer to the seller at the lowest possible cost to themselves, the buyers efficiently use inspection contingencies to share the risk with the seller. In the free market equilibrium, the inspection choices d^* maximize $V(d_i, v_i, r_i)$ over d_i . The policy distorts the effective valuation of every buyer i with $r_i < r^*$, who would waive inspections absent the policy. Namely, $\varphi(r) := V^*(v_i, r_i) - V(1, v_i, r_i) = \zeta(2 - \zeta)\sigma^2 \max\{r^* - r, 0\}$ is the (downward) distortion. Therefore, the policy decreases total surplus realization by realization. The distortion in effective valuations also causes seller surplus to decrease.

Effect on aggregate buyer surplus. Given that buyers are on average less wealthy than sellers of real estate, buyer surplus is a compelling welfare criterion.¹⁸ Proposition 4 shows that under certain conditions, when buyers with higher valuations tend to have (weakly) lower risk aversion, the policy reduces aggregate buyer surplus. On the other hand, a *ban* of inspections raises aggregate buyer surplus.

¹⁷Both inequalities are strict in some cases. $M^p < M^*$ if and only if $r_j < r^*$ for every j with $V^*(v_j, r_j) = M^*$. $\Pr(\tilde{b}^p(M^p) \leq x) > \Pr(\tilde{b}^*(M^*) \leq x)$ for $x \in (\tilde{b}^p(V(1, \underline{v}, \min\{r^*, \bar{r}\})), \tilde{b}^*(V^*(\bar{v}, \underline{r})))$.

¹⁸Indeed, the Massachusetts "Regulation 760 CMR 74.00: Residential home inspection waivers" "is intended to protect the health, safety, and welfare of homebuyers and consumers" (Massachusetts EOHL, 2025b).

Proposition 4. Assume v_i and r_i are negatively affiliated and $f_{v|r}(\cdot|r)$ is log-concave for every r . Then, the no-waiver policy FOSD-decreases aggregate buyer surplus: For every x , $\Pr(M^P - \tilde{b}^P(M^P) \leq x) \geq \Pr(M^* - \tilde{b}^*(M^*) \leq x)$. Conversely, a ban of inspections FOSD-increases aggregate buyer surplus if $r^* < \bar{r}$.¹⁹

Because the game reduces to a first-price auction, in the free market equilibrium, the expected winning bid $\mathbb{E}[\tilde{b}^*(M^*)]$ equals the expected second-highest effective valuation V^* . Similarly, under the policy, the expected winning bid $\mathbb{E}[\tilde{b}^P(M^P)]$ equals the expected second-highest effective valuation $V(1, v_i, r_i)$. Therefore, the effect of the policy on aggregate buyer surplus is pinned down by its effect on the expected difference between the two highest effective valuations.

The policy affects this difference as follows. It reduces the sensitivity of every risk-tolerant buyer's effective valuation to his private information. Namely, absent the policy, when $r_i < r^*$, the slope of buyer i 's effective valuation $V^*(v_i, r_i)$ with respect to his risk aversion r_i is $\partial V^*(v_i, r_i)/\partial r_i = -\sigma^2$. Under the no-waiver policy, it is $\partial V(1, v_i, r_i)/\partial r_i = -(1 - \zeta)^2 \sigma^2$, so that $|\partial V(1, v_i, r_i)/\partial r_i| < |\partial V^*(v_i, r_i)/\partial r_i|$. For $r_i \geq r^*$, the sensitivity of buyer i 's effective valuation to r_i does not change with the policy. Therefore, the policy decreases the dispersion of risk-tolerant buyers' effective valuations. Thus, to the extent that the more risk-tolerant buyers tend to have the highest effective valuations, the policy will tend to decrease the expected difference between the two highest effective valuations.

Indeed, risk aversion enters buyers' effective valuations negatively, so risk-tolerant buyers tend to have the highest effective valuations. When v_i and r_i are negatively affiliated (which includes independence), a risk-tolerant buyer i with $r_i < r^*$ also tends to have a high valuation v_i for the object, so the probability that the two highest effective valuations belong to risk-tolerant buyers is high enough that the policy reduces aggregate buyer surplus.

By a similar logic, a *ban* of inspections increases aggregate buyer surplus. Therefore, perhaps counterintuitively, a buyer-surplus-maximizing policymaker should want to *ban* buyers from performing inspections. Conversely, a no-waiver policy that protects the buyers' right to inspect is actually harmful to buyer surplus.

Negative affiliation is intuitively appealing. Buyer wealth heterogeneity will tend to generate negative affiliation between v_i and r_i if wealthier buyers tend to be less risk-averse—consistent with empirical evidence (see e.g., Guiso and Paiella, 2008; Paravisini et al., 2017)—and have a higher willingness to pay for the object absent risk.

¹⁹ v_i and r_i are *negatively affiliated* if $f_{vr}(v, r)f_{vr}(v', r') \leq f_{vr}(v, r')f_{vr}(v', r)$ for all $v \leq v'$ and $r \leq r'$, which holds with equality if v_i and r_i are stochastically independent (see Milgrom and Weber (1982, p. 1098) and Lehmann (1966, p. 1151)). In expectation, the no-waiver policy decreases aggregate buyer surplus: $\mathbb{E}[M^P - \tilde{b}^P(M^P)] < \mathbb{E}[M^* - \tilde{b}^*(M^*)]$. In expectation, a ban increases aggregate buyer surplus if $r^* < \bar{r}$. Like the no-waiver policy (Proposition 3), a ban FOSD-decreases total and seller surplus (Step 7 of the proof of this proposition).

Distributional effect on buyer surplus. Of course, to the extent that wealth inequality is partly behind risk aversion heterogeneity, an inequality-averse policymaker may not weight all buyers' welfare equally. Namely, between two buyers i and j with the same valuation (i.e., $v_i = v_j$), the policymaker's objective function will assign buyer i 's payoff a higher weight than buyer j 's if $r_i > r_j$. Therefore, it is important to study how the policy's effect on aggregate buyer surplus is distributed across buyers. Denote by $\Delta u^p(v_i, r_i) := u^p(v_i, r_i) - u^*(v_i, r_i)$ the effect of the policy on buyer i 's interim expected surplus, and by $r^\varphi := \min\{r^*, \bar{r}\}$ the lowest risk aversion parameter at which the policy's effect $\varphi(r_i)$ on buyer i 's effective valuation is minimal. Proposition 5 studies the heterogeneous effects of the policy on buyers, which Figure 3 presents. This result suggests that a policymaker who gives less wealthy (and thus more risk-averse) buyers more weight than wealthier ones may find the no-waiver policy beneficial.

Proposition 5. The policy

- (i) harms risk-tolerant buyers: $\Delta u^p(v_i, r_i) < 0$ if and only if $r_i < r^\dagger(V^*(v_i, r_i))$;
- (ii) benefits risk-averse buyers with high valuations: $\Delta u^p(v_i, r_i) > 0$ if and only if $r_i > r^\dagger(V^*(v_i, r_i))$ and $V^*(v_i, r_i) > V^*(\underline{v}, r^\varphi)$; and
- (iii) leaves risk-averse buyers with low valuations unaffected: $\Delta u^p(v_i, r_i) = 0$ if and only if $r_i = r^\dagger(V^*(v_i, r_i))$ or $V^*(v_i, r_i) \leq V^*(\underline{v}, r^\varphi)$

for some continuous $r^\dagger : [V^*(\underline{v}, \bar{r}), V^*(\bar{v}, \underline{r})] \rightarrow (\underline{r}, r^\varphi]$.²⁰

The effect on buyer i 's interim expected surplus can be decomposed as follows:

$$\Delta u^p(v_i, r_i) = \underbrace{\int_{V(1, \underline{v}, \bar{r})}^{V^*(v_i, r_i)} (F_V^p(t)^{N-1} - F_V^*(t)^{N-1}) dt}_{\geq 0; \text{ gain from decreased competition}} - \underbrace{\int_{V(1, v_i, r_i)}^{V^*(v_i, r_i)} F_V^p(t)^{N-1} dt}_{\geq 0; \text{ loss from effective valuation distortion}}.$$

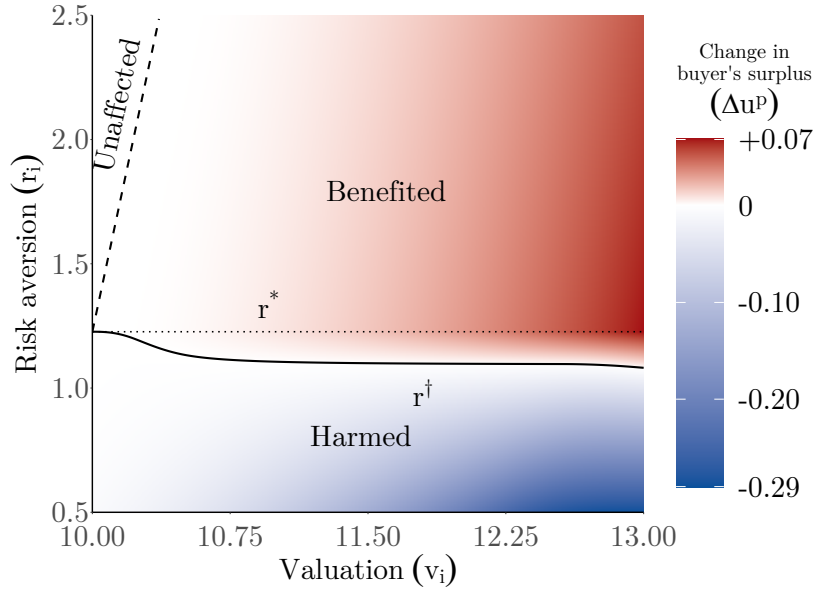
The first term is buyer i 's gain from decreased competition. If we hold his effective valuation fixed at its free market level $V^*(v_i, r_i)$, the buyer's surplus tends to increase thanks to decreased competition from other buyers, whose effective valuations are distorted downward, so that $F_V^p(t) \geq F_V^*(t)$ for every t . This gain is equal to zero if and only if $V^*(v_i, r_i) \leq V(1, \underline{v}, r^\varphi)$, since $F_V^p(t) > F_V^*(t)$ if and only if $t \in (V(1, \underline{v}, r^\varphi), V^*(\bar{v}, \underline{r}))$. The second term is buyer i 's loss from the distortion in his effective valuation. This loss is equal to zero if and only if $r_i \geq r^*$.

Due to the policy, risk-averse buyers (i.e., with $r_i \geq r^*$) face less competition from the less risk-averse buyers, and therefore either benefit from the policy or are unaffected by it. Any buyer i with $r_i \geq r^*$ but a valuation v_i low enough (i.e., with a free market

²⁰If $r^\varphi = \bar{r}$, then $V^*(v_i, r_i) \leq V^*(\underline{v}, r^\varphi)$ if and only if $(v_i, r_i) = (\underline{v}, \bar{r})$, so the set of unaffected buyers has measure zero.

effective valuation $V^*(v_i, r_i) \leq V^*(\underline{v}, r^*)$ is unaffected by the policy. For such a buyer, both the gain from decreased competition and the loss from effective valuation distortion are zero. This is because the policy does not distort (i) his effective valuation or (ii) the effective valuation of any buyer with a lower effective valuation than his, and (iii) the policy does not change the set of buyers with lower effective valuations than his. Therefore, neither his effective valuation nor the competition he faces changes, so his interim expected surplus is unaffected by the policy.²¹ Any buyer i with $r_i \geq r^*$ and a high enough valuation v_i benefits from the policy: His effective valuation is not distorted, and he gains from decreased competition because the effective valuations of some of the buyers he competes with are distorted. Any buyer i who would waive inspections in the free market equilibrium (i.e., with $r_i < r^*$) but whose risk aversion is not too low also benefits from the policy. Although the policy reduces his effective valuation, it also dampens the competition he faces from more risk-tolerant buyers. Last, the policy harms the most risk-tolerant buyers, whose effective valuations are distorted the most.

Figure 3: Heterogeneous no-waiver policy effects on buyers' surplus



Note: The solid curve is the threshold of Proposition 5. The dashed curve outlines the set of unaffected buyers. $N = 3$, $\zeta = 0.4$, $r_S = 1$, $\kappa_B = \kappa_S = 0.25$, $v_i \sim U[10, 13]$, $r_i \sim U[0.5, 2.5]$, $\mu = 2$, and $\sigma^2 = 0.8$ with $\bar{c} = 6$.

3.1.4 The stated-share policy

Consider a policy requiring offers to specify not only the price b_i and the contingency term d_i but also the share $\lambda_i \in [0, 1]$ of the cost c that the seller will bear (if the offer is contingent).

²¹In fact, for every realization of $(v_i, r_i)_{i \in N}$, his realized surplus under the policy is the same as in the free market equilibrium.

The seller's payoff from accepting offer (b_i, d_i, λ_i) is the effective bid $\tilde{b}(b_i, d_i, \lambda_i) := b_i - d_i[\kappa_S + \lambda_i\mu + \lambda_i^2 r_S \sigma^2]$. A buyer i who submits offer (b_i, d_i, λ_i) and wins earns payoff $v_i - d_i\kappa - \mu - [(1 - \lambda_i d_i)^2 r_i + \lambda_i^2 d_i r_S] \sigma^2 - \tilde{b}(b_i, d_i, \lambda_i)$. Buyer i 's effective valuation is now $V^{\text{ss}}(v_i, r_i) := \max\{V(0, v_i, r_i), v_i - \kappa - \mu - (r_i r_S) \sigma^2 / (r_i + r_S)\}$ with CDF $F_V^{\text{ss}}(s) := \int_{\underline{r}}^{\bar{r}} \int_{\underline{v}}^{\bar{v}} \mathbf{I}\{V^{\text{ss}}(v, r) \leq s\} f_{vr}(v, r) dv dr$. $\zeta^{\text{ss}}(r_i) := r_i / (r_i + r_S)$ is the share buyer i specifies in his equilibrium offer.²² Denote by r^{ss} the larger root of $r^2 \sigma^2 = \kappa(r + r_S)$, so that $V^{\text{ss}}(v_i, r_i) > V(0, v_i, r_i)$ if and only if $r_i > r^{\text{ss}}$. Denote by $\omega^{\text{ss}}(r) := \kappa_S + \zeta^{\text{ss}}(r)\mu + \zeta^{\text{ss}}(r)^2 r_S \sigma^2$ the seller's cost of a contingency when she bears share $\zeta^{\text{ss}}(r)$ of the repair cost c .

Proposition 6. Under the stated-share policy, there is a unique symmetric monotone equilibrium: Each buyer i submits offer $(b^{\text{ss}}(v_i, r_i), d^{\text{ss}}(r_i), \zeta^{\text{ss}}(r_i))$ with $d^{\text{ss}}(r_i) = \mathbf{I}\{r_i > r^{\text{ss}}\}$ and the price $b^{\text{ss}}(v_i, r_i) = \tilde{b}^{\text{ss}}(V^{\text{ss}}(v_i, r_i)) + d^{\text{ss}}(r_i)\omega^{\text{ss}}(r_i)$, where $\tilde{b}^{\text{ss}}(s) := s - \int_{V^{\text{ss}}(\underline{v}, \bar{r})}^s F_V^{\text{ss}}(t)^{N-1} dt / F_V^{\text{ss}}(s)^{N-1}$. The buyer with the highest $V^{\text{ss}}(v_i, r_i)$ wins the object.²³

Denote by $M^{\text{ss}} := \max_i V^{\text{ss}}(v_i, r_i)$ the winning buyer's effective valuation. The seller's equilibrium realized surplus is $\tilde{b}^{\text{ss}}(M^{\text{ss}})$. The winning buyer's equilibrium realized surplus is $M^{\text{ss}} - \tilde{b}^{\text{ss}}(M^{\text{ss}})$. M^{ss} is the equilibrium realized total surplus. Buyer i 's interim expected surplus is $u^{\text{ss}}(v_i, r_i) := \int_{V^{\text{ss}}(\underline{v}, \bar{r})}^{V^{\text{ss}}(v_i, r_i)} F_V^{\text{ss}}(t)^{N-1} dt$. Assume in this section that $r^{\text{ss}} < \bar{r}$, so that the policy has an effect.

Effect on total and seller surplus. Proposition 7 shows that the policy causes an FOSD-increase in total surplus and seller surplus.

Proposition 7. The policy FOSD-increases

- (i) total surplus: For every realization $(v_i, r_i)_{i \in \mathcal{N}}$ of the buyers' types, $M^{\text{ss}} \geq M^*$; and
- (ii) seller surplus: For every x , $\Pr(\tilde{b}^{\text{ss}}(M^{\text{ss}}) \leq x) \leq \Pr(\tilde{b}^*(M^*) \leq x)$.

Also, $r^{\text{ss}} \leq r^*$.²⁴

The policy raises the effective valuation of every buyer i with $r_i > r^{\text{ss}}$ and $\zeta^{\text{ss}}(r_i) \neq \zeta$ by improving risk-sharing. Namely, it changes buyer i 's effective valuation by

$$V^{\text{ss}}(v_i, r_i) - V^*(v_i, r_i) = \begin{cases} 0 & \text{if } r_i \leq r^{\text{ss}}, \\ r_i \sigma^2 - \kappa - r_i r_S \sigma^2 / (r_i + r_S) > 0 & \text{if } r^{\text{ss}} < r_i < r^*, \\ (r_i + r_S)(\zeta - \zeta^{\text{ss}}(r_i))^2 \sigma^2 & \text{if } r_i \geq r^*. \end{cases}$$

²²For $r_i = r_S = 0$, read $\zeta^{\text{ss}}(0) = 0$ and $r_i r_S / (r_i + r_S) = 0$.

²³The equilibrium strategy is unique (in the class of symmetric monotone equilibria), up to the share stated in a non-contingent offer, for every (v_i, r_i) such that $r_i \neq r^{\text{ss}}$ and $(v_i, r_i) \neq (\underline{v}, \bar{r})$.

²⁴The three inequalities are strict in some cases. $M^{\text{ss}} > M^*$ if and only if $r_i > r^{\text{ss}}$ and $\zeta \neq \zeta^{\text{ss}}(r_i)$ for every i with $V^{\text{ss}}(v_i, r_i) = M^{\text{ss}}$. $\Pr(\tilde{b}^{\text{ss}}(M^{\text{ss}}) \leq x) < \Pr(\tilde{b}^*(M^*) \leq x)$ for $x \in (V^*(\underline{v}, \bar{r}), \tilde{b}^{\text{ss}}(V^{\text{ss}}(\bar{v}, r)))$. If $r^* > 0$, $r^{\text{ss}} < r^*$ if and only if $\zeta \neq \zeta^{\text{ss}}(r^*)$.

Therefore, $M^{\text{ss}} \geq M^*$ realization by realization. As a result, seller surplus also increases. Also, due to more efficient risk-sharing, inspection contingencies are used more often (i.e., $r^{\text{ss}} \leq r^*$) than in the free market equilibrium. In the free market, a buyer i with $r_i \in (r^{\text{ss}}, r^*)$ waives an inspection that would be worth performing if risk were efficiently shared post-inspection.

We conclude that a policy that enriches the contract space—instead of restricting it (by preventing buyers from committing to whether they will perform an inspection)—would improve risk-sharing, not only by allowing buyers to commit to whether they will perform an inspection but also by requiring them to pre-specify how the repair cost revealed by the inspection will be shared with the seller. Of course, this would make it even more important that the parties can agree on a trustworthy inspector—which is also important in the status quo, where the division of costs is not agreed upon in advance.

Effect on aggregate buyer surplus. Propositions 8 and 9 study buyer surplus under two alternative (and mutually exclusive) conditions.

Assumption 3. (a) $r^* \geq \bar{r}$ or $\zeta \leq \zeta^{\text{ss}}(\max\{r^*, \underline{r}\})^{25}$; (b) $r^* \leq \underline{r}$ and $\zeta \geq \zeta^{\text{ss}}(\bar{r})$.

Under assumption 3(a), either every buyer waives inspections in the free market ($r^* \geq \bar{r}$) or the post-inspection bargaining power of every buyer who submits a contingent offer is inefficiently low ($\zeta \leq \zeta^{\text{ss}}(r_i)$ for every $r_i \geq \max\{r^*, \underline{r}\}$). Then, the gain $V^{\text{ss}}(v_i, r_i) - V^*(v_i, r_i)$ is non-decreasing in r_i on $[\underline{r}, \bar{r}]$. Conversely, under assumption 3(b), every buyer submits a contingent offer in the free market, and his post-inspection bargaining power is inefficiently high. Then, the gain $V^{\text{ss}}(v_i, r_i) - V^*(v_i, r_i)$ is non-increasing. Proposition 8 shows that, under the same conditions as in Proposition 4, the policy FOSD-decreases aggregate buyer surplus under assumption 3(a) and FOSD-increases it under assumption 3(b).

Proposition 8. Assume v_i and r_i are negatively affiliated and $f_{v|r}(\cdot|r)$ is log-concave for every r . Under assumption 3(a), the policy FOSD-decreases aggregate buyer surplus: For every x , $\Pr(M^{\text{ss}} - \tilde{b}^{\text{ss}}(M^{\text{ss}}) \leq x) \geq \Pr(M^* - \tilde{b}^*(M^*) \leq x)$. Under assumption 3(b), the policy FOSD-increases aggregate buyer surplus: For every x , $\Pr(M^{\text{ss}} - \tilde{b}^{\text{ss}}(M^{\text{ss}}) \leq x) \leq \Pr(M^* - \tilde{b}^*(M^*) \leq x)$.²⁶

Assumption 3(a) holds when $\kappa > 0$ and the buyers' post-inspection bargaining power ζ is low. Under assumption 3(b), every buyer submits a contingent offer in the free market, with the seller responsible for a higher-than-efficient share of the repair cost. Then, the policy makes all buyers' effective valuations more sensitive to their risk aversion, thereby increasing aggregate buyer surplus.

²⁵If $\kappa = r_S = \underline{r} = 0$, read $\zeta^{\text{ss}}(0)$ here as 1, the share every buyer with $r_i > 0$ specifies.

²⁶Aggregate buyer surplus decreases strictly in expectation under assumption 3(a), and increases strictly in expectation under assumption 3(b).

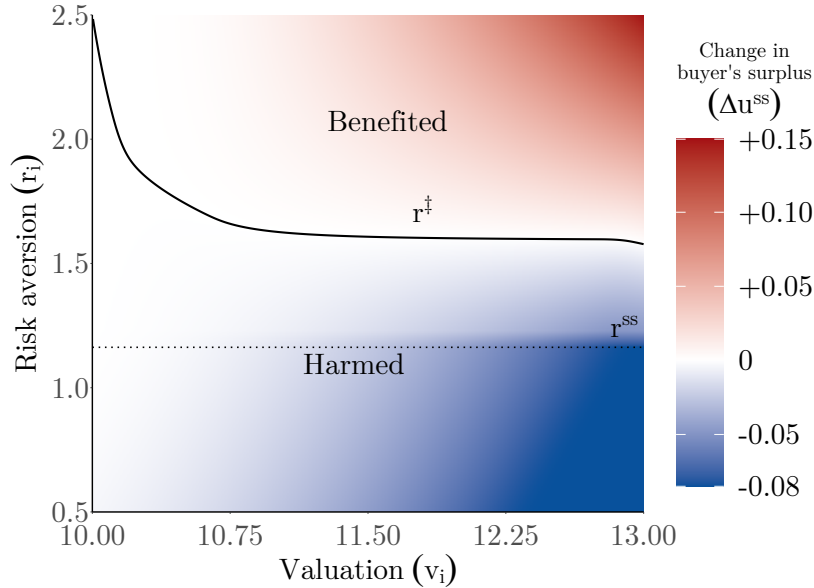
Distributional effect on buyer surplus. Denote by $\Delta u^{\text{ss}}(v_i, r_i) := u^{\text{ss}}(v_i, r_i) - u^*(v_i, r_i)$ the effect of the policy on buyer i 's interim expected surplus. Proposition 9 studies the heterogeneous effects of the policy on buyers, which Figure 4 presents.

Proposition 9. Under assumption 3(a) (resp. 3(b)), the policy

- (i) harms risk-tolerant (resp. risk-averse) buyers: $\Delta u^{\text{ss}}(v_i, r_i) < 0$ if and only if $r_i < r^\dagger(V^*(v_i, r_i))$ (resp. $r_i > r^\dagger(V^*(v_i, r_i))$);
- (ii) benefits risk-averse (resp. risk-tolerant) buyers: $\Delta u^{\text{ss}}(v_i, r_i) > 0$ if and only if $r_i > r^\dagger(V^*(v_i, r_i))$ (resp. $r_i < r^\dagger(V^*(v_i, r_i))$); and
- (iii) leaves the remaining buyers unaffected: $\Delta u^{\text{ss}}(v_i, r_i) = 0$ if and only if $r_i = r^\dagger(V^*(v_i, r_i))$

for some continuous $r^\dagger : [V^*(\underline{v}, \bar{r}), V^*(\bar{v}, \underline{r})] \rightarrow (\max\{r^{\text{ss}}, \underline{r}\}, \bar{r}]$.²⁷

Figure 4: Heterogeneous stated-share policy effects on buyers' surplus



Note: The solid curve is the threshold of Proposition 9. The dotted line is r^{ss} . The parameters are those of Figure 3, under which assumption 3(a) holds.

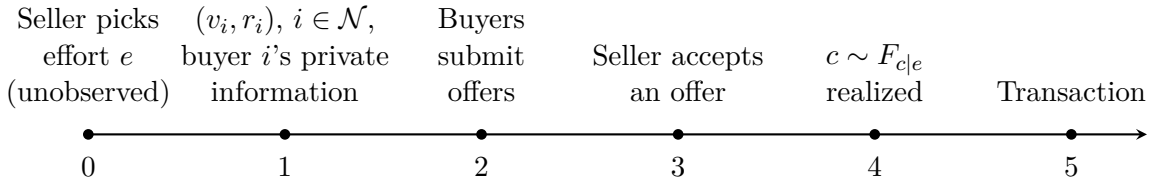
3.2 Moral hazard: inspections and maintenance

Figure 5 presents the timeline of the game. The game is the baseline game with one added stage. Before the informal auction, the seller privately chooses $e \in \{0, 1\}$, that is, whether to exert effort to maintain the object (i.e., $e = 1$) or to shirk (i.e., $e = 0$). Effort affects

²⁷Whether or not assumption 3 holds, the set of unaffected buyers has measure zero, and $\Delta u^{\text{ss}}(v_i, r_i) < 0$ if $r_i \leq r^{\text{ss}}$.

the distribution of the repair cost c . Denote by $\mu_e := \mathbb{E}[c \mid e]$ and $\sigma_e^2 := \text{Var}(c \mid e)$ the mean and variance of c when the seller chooses effort e . $\Delta_\mu := \mu_0 - \mu_1 > 0$ is the effect of effort on the mean. $\Delta_{\sigma^2} := \sigma_0^2 - \sigma_1^2 \geq 0$ is the effect on the variance, and $\sigma_1^2 > 0$. At either effort level, $F_{c|e}$ has a positive density on $(0, \bar{c})$. Assumption 1 is assumed to hold for (μ_0, σ_0^2) , hence for (μ_1, σ_1^2) as well since $\mu_1 < \mu_0$ and $\sigma_1^2 \leq \sigma_0^2$. Effort (i.e., $e = 1$) costs the seller $\theta > 0$. For simplicity, we focus on pure-strategy equilibria, in which the seller does not randomize her choice of e , and in which, in every continuation game starting from stage 1, buyers play the strategies described in Propositions 1 and 2 in the free market and under the no-waiver policy, respectively.

Figure 5: Game with moral hazard: timeline



3.2.1 The free market equilibrium

Assume buyers believe the seller has chosen effort $\hat{e} \in \{0, 1\}$ in stage 0 and that she has actually chosen effort $e \in \{0, 1\}$. The seller's payoff from accepting offer (b_i, d_i) is $\pi_S(b_i, d_i; r_S|e) = \tilde{b}(b_i, d_i|e)$, where $\tilde{b}(b_i, d_i|e) := b_i - d_i\omega_e$ is the actual effective bid and $\omega_e := \kappa_S + \zeta\mu_e + \zeta^2r_S\sigma_e^2$. A buyer i who submits offer (b_i, d_i) and wins expects payoff $\pi_B(b_i, d_i; v_i, r_i|\hat{e}) = V(d_i, v_i, r_i|\hat{e}) - \tilde{b}(b_i, d_i|\hat{e})$, where

$$V(d_i, v_i, r_i | \hat{e}) := \begin{cases} v_i - \mu_{\hat{e}} - r_i \sigma_{\hat{e}}^2 & \text{if } d_i = 0, \\ v_i - \kappa - \mu_{\hat{e}} - r_i (1 - \zeta)^2 \sigma_{\hat{e}}^2 - \zeta^2 \sigma_{\hat{e}}^2 r_S & \text{if } d_i = 1. \end{cases}$$

Denote buyer i 's effective valuation in the free market under belief $\hat{e}$ by

$$V^*(v_i, r_i | \hat{e}) := \max_{d_i \in \{0,1\}} V(d_i, v_i, r_i | \hat{e}) = \begin{cases} V(0, v_i, r_i | \hat{e}) & \text{if } r_i < r^*(\sigma_e^2), \\ V(1, v_i, r_i | \hat{e}) & \text{if } r_i \geq r^*(\sigma_e^2), \end{cases}$$

where $r^*(\sigma_e^2) := \kappa/[\zeta(2-\zeta)\sigma_e^2] + \zeta r_S/(2-\zeta)$. Denote by $F_V^*(s|\hat{e}) := \int_{\underline{r}}^{\bar{r}} \int_{\underline{v}}^{\bar{v}} \mathbf{I}\{V^*(v, r|\hat{e}) \leq s\} f_{vr}(v, r) dv dr$ its CDF. By Proposition 1, given belief $\hat{e}$, each buyer i submits offer $(b^*(v_i, r_i|\hat{e}), d^*(r_i|\hat{e}))$ with $d^*(r_i|\hat{e}) = \mathbf{I}\{r_i \geq r^*(\sigma_e^2)\}$ and $b^*(v_i, r_i|\hat{e}) = \tilde{b}^*(V^*(v_i, r_i|\hat{e})|\hat{e}) + d^*(r_i|\hat{e})\omega_{\hat{e}}$, where $\tilde{b}^*(s|\hat{e}) := s - \int_{V^*(v, \bar{r}|\hat{e})}^s F_V^*(t|\hat{e})^{N-1} dt / F_V^*(s|\hat{e})^{N-1}$.

If the seller maintains the object, her payoff increases by $\Delta_\omega := \omega_0 - \omega_1 = \zeta \Delta_\mu + \zeta^2 r_S \Delta_{\sigma^2} > 0$ (relative to not maintaining it) if the offer she accepts includes an inspection. After choosing effort e , the seller earns payoff $\tilde{b}(b^*(v_i, r_i | \hat{e}), d^*(r_i | \hat{e}) | e) = b^*(v_i, r_i | \hat{e}) -$

$d^*(r_i|\hat{e})\omega_e = \tilde{b}^*(V^*(v_i, r_i|\hat{e})|\hat{e}) + d^*(r_i|\hat{e})(\omega_{\hat{e}} - \omega_e)$ from accepting buyer i 's offer, so her benefit from maintaining the object is $\Gamma(\hat{e})$, where

$$\Gamma(\hat{e}) := \mathbb{E} \left[\max_i \left(\tilde{b}^*(V^*(v_i, r_i|\hat{e})|\hat{e}) + d^*(r_i|\hat{e})(\omega_{\hat{e}} - \omega_1) \right) \right] \\ - \mathbb{E} \left[\max_i \left(\tilde{b}^*(V^*(v_i, r_i|\hat{e})|\hat{e}) + d^*(r_i|\hat{e})(\omega_{\hat{e}} - \omega_0) \right) \right],$$

while maintenance costs her θ . If we denote by $L(\sigma_{\hat{e}}^2) := \max_{i:r_i < r^*(\sigma_{\hat{e}}^2)} \tilde{b}^*(V^*(v_i, r_i|\hat{e})|\hat{e}) - \max_{i:r_i \geq r^*(\sigma_{\hat{e}}^2)} \tilde{b}^*(V^*(v_i, r_i|\hat{e})|\hat{e})$ the (random) difference between the seller's payoff from accepting the best non-contingent offer and her payoff from accepting the best contingent one, and by $\rho(\sigma_{\hat{e}}^2) := \Pr(L(\sigma_{\hat{e}}^2) < 0)$ the probability that the winning offer is contingent when the buyers' beliefs are consistent (i.e., $e = \hat{e}$),²⁸ $\Gamma(\hat{e})$ can be equivalently written as

$$\Gamma(0) = \underbrace{\rho(\sigma_0^2)\Delta_{\omega}}_{\text{direct remediation cost savings from maintenance}} + \underbrace{\mathbb{E}[\max\{\Delta_{\omega} - L(\sigma_0^2), 0\}\mathbf{I}\{L(\sigma_0^2) > 0\}]}_{\text{option value of maintenance } \in [0, \Delta_{\omega} \Pr(0 < L(\sigma_0^2) < \Delta_{\omega})]}, \\ \Gamma(1) = \underbrace{\rho(\sigma_1^2)\Delta_{\omega}}_{\text{direct remediation cost savings from maintenance}} - \underbrace{\mathbb{E}[\max\{\Delta_{\omega} + L(\sigma_1^2), 0\}\mathbf{I}\{L(\sigma_1^2) < 0\}]}_{\text{option value of maintenance } \in [0, \Delta_{\omega} \Pr(-\Delta_{\omega} < L(\sigma_1^2) < 0)]},$$

where Δ_{ω} is the seller's savings in remediation costs when the winning offer is contingent. Lemma 2 provides bounds on $\Gamma(\hat{e})$ that follow from this decomposition.

Lemma 2. For every $\hat{e} \in \{0, 1\}$, $[1 - F_r(r^*(\sigma_{\hat{e}}^2))]^N \Delta_{\omega} \leq \Pr(L(\sigma_{\hat{e}}^2) \leq -\hat{e}\Delta_{\omega})\Delta_{\omega} \leq \Gamma(\hat{e}) \leq \Pr(L(\sigma_{\hat{e}}^2) < (1 - \hat{e})\Delta_{\omega})\Delta_{\omega} \leq [1 - F_r(r^*(\sigma_{\hat{e}}^2))]^N \Delta_{\omega}$.²⁹

The decomposition also clarifies how the buyers' beliefs $\hat{e}$ about maintenance affect the seller's incentive $\Gamma(\hat{e})$ to maintain the object. This incentive hinges upon the probability that the offer she accepts is contingent. Buyers' beliefs about maintenance affect this probability through three forces.

Two forces act directly on the probability $\rho(\sigma_{\hat{e}}^2)$ that the winning offer is contingent when we hold the seller's offer-acceptance strategy fixed at the strategy she would optimally follow if she had actually chosen $e = \hat{e}$. First, by decreasing the repair-cost risk, the belief that the seller has maintained the object lowers the share of contingent offers (because $\sigma_1^2 \leq \sigma_0^2$, $r^*(\sigma_1^2) \geq r^*(\sigma_0^2)$, and thus $F_r(r^*(\sigma_1^2)) \geq F_r(r^*(\sigma_0^2))$). This tends to make the probability $\rho(\sigma_1^2)$ that the winning offer is contingent lower when the buyers believe the seller has maintained the object than the probability $\rho(\sigma_0^2)$ when they believe she has not. Due to this force, when buyers believe that the seller has maintained the object,

²⁸The maximum over an empty set of buyers is defined as $-\infty$. Both $L(\sigma_{\hat{e}}^2)$ and $\rho(\sigma_{\hat{e}}^2)$ depend on the belief $\hat{e}$ only through $\sigma_{\hat{e}}^2$; $\mu_{\hat{e}}$ shifts every effective bid by the same amount.

²⁹The inequalities $\Pr(L(\sigma_{\hat{e}}^2) \leq -\hat{e}\Delta_{\omega})\Delta_{\omega} \leq \Gamma(\hat{e}) \leq \Pr(L(\sigma_{\hat{e}}^2) < (1 - \hat{e})\Delta_{\omega})\Delta_{\omega}$ hold strictly if and only if $\underline{r} < r^*(\sigma_{\hat{e}}^2) < \bar{r}$. In particular, (i) $\Gamma(\hat{e}) = \Delta_{\omega}$ if $r^*(\sigma_{\hat{e}}^2) \leq \underline{r}$, (ii) $\Gamma(\hat{e}) \in (0, \Delta_{\omega})$ if $r^*(\sigma_{\hat{e}}^2) \in (\underline{r}, \bar{r})$, and (iii) $\Gamma(\hat{e}) = 0$ if $r^*(\sigma_{\hat{e}}^2) \geq \bar{r}$.

the seller's incentives to maintain in the first place are weak. Second, by decreasing risk, the belief that the seller has maintained the object also decreases the sensitivity of effective valuations to risk aversion, thereby potentially making it more likely that a more risk-averse buyer—who inspects—wins. This force therefore pushes in the *opposite* direction: When buyers believe that the seller has maintained the object, the seller's incentives to maintain are strong.

The third force is due to the option value of maintenance. Buyers compensate the seller with ω_e for including an inspection contingency, while the seller prices the contingency at ω_e . When buyers believe maintenance has not taken place (i.e., $\hat{e} = 0$), maintaining the object makes a contingency cost her Δ_ω less than the compensation ω_0 offered by the buyers for the contingency. Thus, if the seller maintains the object and it turns out that $L(\sigma_0^2) \in (0, \Delta_\omega)$, she will then accept the best contingent offer instead of the best non-contingent one (which she would accept if she had chosen $e = 0$). Conversely, when buyers believe the seller has maintained the object, skipping maintenance makes a contingency cost her $\Delta_\omega \equiv \omega_0 - \omega_1$ more than the compensation ω_1 offered by the buyers for the contingency. In other words, buyers undercompensate her for the contingency, which makes her favor non-contingent offers instead. Thus, if the seller forgoes maintenance and it turns out that $L(\sigma_1^2) \in (-\Delta_\omega, 0)$, she will then accept the best non-contingent offer instead of the best contingent one (which she would accept if she had chosen $e = 1$). Due to the third force, the seller's incentives to maintain in the first place tend to be weak when buyers believe that the seller has maintained the object.

Proposition 10 describes the equilibrium effort choices.

Proposition 10. There is a free market equilibrium where the seller maintains (resp. does not maintain) the object if and only if $\Gamma(1) \geq \theta$ (resp. $\Gamma(0) \leq \theta$).

This means that for intermediate maintenance costs (i.e., when $\theta \in [\Gamma(0), \Gamma(1)]$), there is equilibrium multiplicity: Both an equilibrium where the seller maintains and an equilibrium where she does not maintain exist. Whether the two equilibria can coexist depends on whether maintenance is “self-sustaining.” Due to the second force described above, when buyers believe that the seller has maintained the object, it tends to become more likely that the winning offer will be contingent, and thus the seller's incentives to maintain are strong. Therefore, when the second force dominates the other two, maintenance of the object and inspection contingencies are strategic complements. As a result, for intermediate maintenance costs, there is equilibrium multiplicity, and maintenance is self-sustaining. When the first and third forces dominate, maintenance of the object and inspection contingencies are strategic substitutes. As a result, the two equilibria cannot coexist.

Lemma 3 provides three alternative sufficient conditions for $\Gamma(1) < \Gamma(0)$ to hold, in

which case the two equilibria cannot coexist.³⁰

Lemma 3. Assume $\underline{r} < r^*(\sigma_1^2)$ and $r^*(\sigma_0^2) < \bar{r}$. Then, $\Gamma(1) < \Gamma(0)$ (i) if $\Delta_{\sigma^2} = 0$, or (ii) if $r^*(\sigma_1^2) \geq \bar{r}$, or (iii) if $(1 - \zeta)^2 \bar{r} + \zeta^2 r_S \leq \underline{r}$.

When $\Delta_{\sigma^2} = 0$, the first and second forces vanish. When $r^*(\sigma_1^2) \geq \bar{r}$, no buyer inspects when he believes the seller has maintained the object, so the second force vanishes. Last, under condition (iii) of Lemma 3, a contingent buyer i 's cost of perceived risk (which he shares with the seller) is lower by $[(1 - \zeta)^2 r_i + \zeta^2 r_S] \Delta_{\sigma^2}$ when he believes the seller has maintained the object than when he believes she has not. On the other hand, a waiving buyer j 's cost of perceived risk is lower by $r_j \Delta_{\sigma^2} \geq \underline{r} \Delta_{\sigma^2} \geq [(1 - \zeta)^2 \bar{r} + \zeta^2 r_S] \Delta_{\sigma^2} \geq [(1 - \zeta)^2 r_i + \zeta^2 r_S] \Delta_{\sigma^2}$ when he believes the seller has maintained the object than when he believes she has not. Therefore, the belief that the seller has maintained the object never gives a contingent buyer a winning advantage over a waiving one.

3.2.2 Equilibrium under the no-waiver policy

As before, consider a policy that prevents buyers from waiving inspections. Assumption 2 is assumed to hold for (μ_1, σ_1^2) , hence for (μ_0, σ_0^2) as well since $\mu_0 > \mu_1$ and $\sigma_0^2 \geq \sigma_1^2$. This implies that in the no-waiver policy equilibrium, every buyer inspects regardless of whether he believes the seller has maintained the object.

Assume buyers believe the seller has chosen effort $\hat{e}$ and that she has actually chosen effort e . Since every buyer inspects if he wins, the seller's payoff from accepting offer b_i is $\pi_S(b_i, 1; r_S | e) = \tilde{b}(b_i, 1 | e) = b_i - \omega_e$, and a buyer i who submits offer b_i and wins expects payoff $\pi_B(b_i, 1; v_i, r_i | \hat{e}) = V(1, v_i, r_i | \hat{e}) - \tilde{b}(b_i, 1 | \hat{e})$. Denote by $F_V^p(s | \hat{e}) := \int_{\underline{r}}^{\bar{r}} \int_{\underline{v}}^{\bar{v}} \mathbf{I}\{V(1, v, r | \hat{e}) \leq s\} f_{vr}(v, r) dv dr$ the CDF of his effective valuation $V(1, v_i, r_i | \hat{e})$. By Proposition 2, given belief $\hat{e}$, each buyer i submits price $b^p(v_i, r_i | \hat{e}) = \tilde{b}^p(V(1, v_i, r_i | \hat{e}) | \hat{e}) + \omega_{\hat{e}}$, where $\tilde{b}^p(s | \hat{e}) := s - \int_{V(1, \underline{v}, \bar{r} | \hat{e})}^s F_V^p(t | \hat{e})^{N-1} dt / F_V^p(s | \hat{e})^{N-1}$. The seller earns payoff $\tilde{b}(b^p(v_i, r_i | \hat{e}), 1 | e) = b^p(v_i, r_i | \hat{e}) - \omega_e = \tilde{b}^p(V(1, v_i, r_i | \hat{e}) | \hat{e}) + \omega_{\hat{e}} - \omega_e$ from accepting buyer i 's offer, so her benefit from maintaining the object is Δ_{ω} , while maintenance costs her θ . Unlike in the free market, the seller's benefit from maintenance is now independent of the buyers' beliefs $\hat{e}$. Proposition 11 describes the equilibrium effort choices.

Proposition 11. Under the no-waiver policy, if $\Delta_{\omega} \neq \theta$, there is a unique equilibrium: The seller maintains (resp. does not maintain) the object if $\Delta_{\omega} > \theta$ (resp. $\Delta_{\omega} < \theta$). When $\Delta_{\omega} = \theta$, both equilibria exist.

Since $\Delta_{\omega} \equiv \zeta \Delta_{\mu} + \zeta^2 r_S \Delta_{\sigma^2}$, the seller maintains the object if $\zeta \Delta_{\mu} + \zeta^2 r_S \Delta_{\sigma^2} > \theta$, which holds when (i) the cost θ of maintenance is low, (ii) buyers have high bargaining

³⁰In that case, if $\theta \in (\Gamma(1), \Gamma(0))$, no pure-strategy equilibrium exists.

power ζ after inspections, (iii) the seller's risk aversion r_S is high, or (iv) the effect Δ_μ of maintenance on the mean repair cost or its effect Δ_{σ^2} on the variance is large.

3.2.3 No-waiver policy effects

Assume throughout this section that $r^*(\sigma_0^2) > \underline{r}$, so that the no-waiver policy has an effect. Then, by Lemma 2, $\Gamma(0) < \Delta_\omega$, and I also assume throughout this section that $\theta \in [\Gamma(0), \Delta_\omega]$.³¹ Then, there exists a free market equilibrium where the seller does not maintain the object (Proposition 10), and under the no-waiver policy, there is a unique equilibrium, in which she does maintain it (Proposition 11). I compare these two equilibria. Later in this section, I discuss equilibrium multiplicity.

In the free market equilibrium, $M_{mh}^* := \max_i V^*(v_i, r_i|0)$ is the winning buyer's effective valuation. The seller's equilibrium realized surplus is $\tilde{b}^*(M_{mh}^*|0)$. The winning buyer's equilibrium realized surplus is $M_{mh}^* - \tilde{b}^*(M_{mh}^*|0)$. M_{mh}^* is the equilibrium realized total surplus. Buyer i 's interim expected surplus is $u_{mh}^*(v_i, r_i) := \int_{V^*(\underline{v}, \bar{r}|0)}^{V^*(v_i, r_i|0)} F_V^*(t|0)^{N-1} dt$.

In the equilibrium under the no-waiver policy, $M_{mh}^P := \max_i V(1, v_i, r_i|1)$ is the winning buyer's effective valuation. The seller's equilibrium realized surplus is $\tilde{b}^P(M_{mh}^P|1) - \theta$. The winning buyer's equilibrium realized surplus is $M_{mh}^P - \tilde{b}^P(M_{mh}^P|1)$. $M_{mh}^P - \theta$ is the equilibrium realized total surplus. Buyer i 's interim expected surplus is $u_{mh}^P(v_i, r_i) := \int_{V(1, \underline{v}, \bar{r}|1)}^{V(1, v_i, r_i|1)} F_V^P(t|1)^{N-1} dt$.

Denote by $\varphi(r|\hat{e}) := \zeta(2 - \zeta)\sigma_e^2 \max\{r^*(\sigma_e^2) - r, 0\}$ the risk-sharing distortion of section 3.1 evaluated at σ_e^2 . The policy changes buyer i 's effective valuation by $\psi(r_i) := V(1, v_i, r_i|1) - V^*(v_i, r_i|0)$, which can be decomposed as

$$\psi(r_i) = \underbrace{V^*(v_i, r_i|1) - V^*(v_i, r_i|0)}_{\substack{>0; \text{ gain from maintenance} \\ \parallel}} - \overbrace{\varphi(r_i|1)}^{\geq 0; \text{ risk-sharing distortion}},$$

$$\Delta_\mu + \begin{cases} r_i \Delta_{\sigma^2} & \text{if } r_i < r^*(\sigma_0^2), \\ \kappa + \zeta^2 r_S \sigma_0^2 + r_i [(1-\zeta)^2 \sigma_0^2 - \sigma_1^2] & \text{if } r^*(\sigma_0^2) \leq r_i < r^*(\sigma_1^2), \\ [(1-\zeta)^2 r_i + \zeta^2 r_S] \Delta_{\sigma^2} & \text{if } r_i \geq r^*(\sigma_1^2). \end{cases}$$

and is non-decreasing in r_i . The first term, $V^*(v, r|1) - V^*(v, r|0)$, captures the gain a buyer would obtain from the decrease in the expected repair cost and in the associated risk due to seller maintenance if the policy induced maintenance without distorting buyers' contingency choices. All buyers' effective valuations increase by the decrease Δ_μ in the expected repair cost. The increase due to the risk reduction depends on whether a buyer inspects (i) both under and absent the policy, (ii) only absent the policy, or (iii) in neither case. The second term captures the downward distortion due to the inefficiencies in risk-sharing introduced by the policy, as in section 3.1. Unlike in section 3.1, the overall effect $\psi(r_i)$ of the policy on buyer i 's effective valuation can be positive (i.e., when the

³¹By Lemma 2, $[1 - F_r(r^*(\sigma_0^2))^N] \Delta_\omega \leq \theta < \Delta_\omega$ is sufficient for $\theta \in [\Gamma(0), \Delta_\omega]$.

first force dominates).

Effect on total and seller surplus. Proposition 12 provides sufficient conditions for the first force to dominate by enough that the increase in effective valuations more than compensates for the maintenance cost. Then, the no-waiver policy FOSD-increases total surplus and seller surplus. Figure 6 presents such an example.

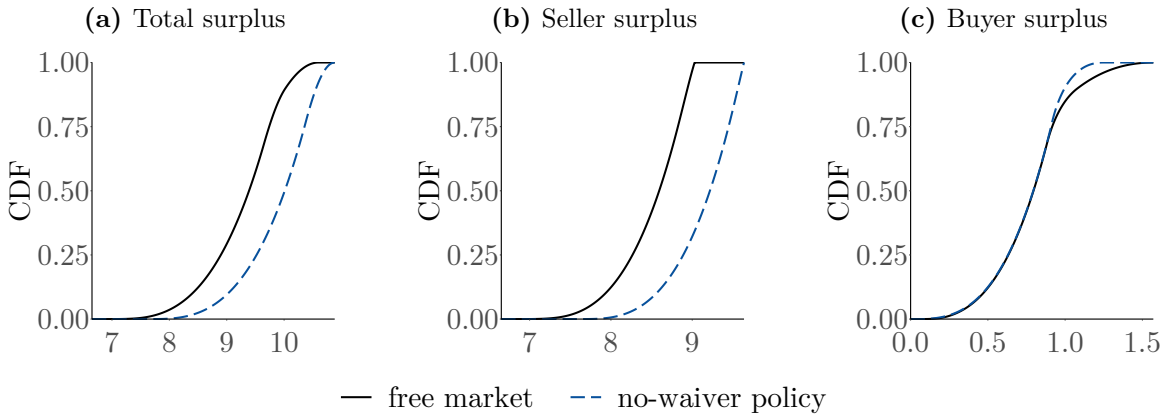
Proposition 12. If $\theta < \psi(\underline{r})$, the policy FOSD-increases

- (i) total surplus: For every $(v_i, r_i)_{i \in \mathcal{N}}$, $M_{\text{mh}}^{\text{P}} - \theta > M_{\text{mh}}^*$; and
- (ii) seller surplus: For every x , $\Pr(\tilde{b}^{\text{P}}(M_{\text{mh}}^{\text{P}}|1) - \theta \leq x) \leq \Pr(\tilde{b}^*(M_{\text{mh}}^*|0) \leq x)$.

If, instead, $\theta > \psi(\bar{r})$, the policy FOSD-decreases total surplus and seller surplus.³²

Notice that $\psi(\underline{r}) = \Delta_\mu + \underline{r}\Delta_{\sigma^2} - \varphi(\underline{r}|1)$. When the inspection cost κ is low and maintenance is highly efficient (i.e., θ is low, Δ_μ is high, and/or Δ_{σ^2} is high), the policy increases total surplus and seller surplus. Sufficiently high buyer risk aversion relative to the seller's (i.e., $\underline{r}$ is high enough compared to r_S) also tends to make the policy welfare-enhancing. This is because, when buyers are sufficiently risk-averse, it is efficient for many of them to include inspection contingencies, and therefore the risk-sharing distortion of the policy is not too large. The policy decreases total surplus and seller surplus in the opposite case.

Figure 6: The no-waiver policy under hidden maintenance, in distribution



Note: Parameters as in Figure 3 with $(\mu_0, \sigma_0^2) = (2, 0.8)$, $(\mu_1, \sigma_1^2) = (1, 0.7)$, $\theta = 0.38$.

³²Given the section's maintained assumption $\theta \in [\Gamma(0), \Delta_\omega]$, for $\theta < \psi(\underline{r})$ to hold, it is necessary that $\Gamma(0) < \psi(\underline{r})$. $\theta < \psi(\underline{r})$ holds for every $\theta \in [\Gamma(0), \Delta_\omega]$ if and only if $\psi(\underline{r}) \geq \Delta_\omega$, or equivalently, $\varphi(\underline{r}|0) \leq (1 - \zeta)\Delta_\mu + (1 - \zeta)^2 \underline{r}\Delta_{\sigma^2}$. For $\theta > \psi(\bar{r})$ to hold, it is necessary that no buyer inspects in the free market (i.e., $r^*(\sigma_0^2) > \bar{r}$).

Effect on aggregate buyer surplus. Proposition 13 shows that, despite inducing efficient maintenance, the no-waiver policy still reduces aggregate buyer surplus when buyers with higher valuations tend to have (weakly) lower risk aversion.

Proposition 13. Assume v_i and r_i are negatively affiliated and $f_{v|r}(\cdot|r)$ is log-concave for every r . Then, the no-waiver policy FOSD-decreases aggregate buyer surplus: For every x , $\Pr(M_{\text{mh}}^{\text{P}} - \tilde{b}^{\text{P}}(M_{\text{mh}}^{\text{P}}|1) \leq x) \geq \Pr(M_{\text{mh}}^* - \tilde{b}^*(M_{\text{mh}}^*|0) \leq x)$.³³

As in section 3.1, the effect of the policy on aggregate buyer surplus is pinned down by its effect on the expected difference between the two highest effective valuations. The same forces identified in section 3.1 tend to make the policy reduce aggregate buyer surplus, and the fact that the policy induces efficient maintenance does not reverse the result. The reduction Δ_μ in the expected repair cost applies to every buyer and is thus competed away. Indeed, the seller reaps the benefits of this reduction, as shown in Proposition 12. The reduction Δ_{σ^2} in repair-cost risk does not work in the buyers' favor either. In the setting of section 3.1, the no-waiver policy reduced the sensitivity of every risk-tolerant buyer's effective valuation to his private information (i.e., his risk aversion) by preventing him from waiving inspections. Since risk-tolerant buyers tend to have the highest effective valuations, the policy decreased aggregate buyer surplus. Now, the no-waiver policy reduces the sensitivity of *every*—not necessarily risk-tolerant—buyer's effective valuation to his risk aversion through an additional force: By inducing maintenance, the policy decreases the repair-cost risk.

Distributional effect on buyer surplus. Denote by $\Delta u_{\text{mh}}^{\text{P}}(v_i, r_i) := u_{\text{mh}}^{\text{P}}(v_i, r_i) - u_{\text{mh}}^*(v_i, r_i)$ the effect of the policy on buyer i 's interim expected surplus, and by $r^\psi := \min\{r \in [\underline{r}, \bar{r}] : \psi(r) = \psi(\bar{r})\}$ the lowest risk aversion parameter at which the policy's effect $\psi(r_i)$ on buyer i 's effective valuation is maximal. If $\Delta_{\sigma^2} > 0$, $r^\psi = \bar{r}$. If $\Delta_{\sigma^2} = 0$, $r^\psi = \min\{r^*(\sigma_0^2), \bar{r}\}$. Proposition 14 studies the heterogeneous effects of the policy on buyers, which Figure 7 presents.

Proposition 14. The no-waiver policy

- (i) harms risk-tolerant buyers: $\Delta u_{\text{mh}}^{\text{P}}(v_i, r_i) < 0$ if and only if $r_i < r^\dagger(V^*(v_i, r_i|0))$;
- (ii) benefits risk-averse buyers with high valuations: $\Delta u_{\text{mh}}^{\text{P}}(v_i, r_i) > 0$ if and only if $r_i > r^\dagger(V^*(v_i, r_i|0))$ and $V^*(v_i, r_i|0) > V^*(\underline{v}, r^\psi|0)$; and
- (iii) leaves risk-averse buyers with low valuations unaffected: $\Delta u_{\text{mh}}^{\text{P}}(v_i, r_i) = 0$ if and only if $r_i = r^\dagger(V^*(v_i, r_i|0))$ or $V^*(v_i, r_i|0) \leq V^*(\underline{v}, r^\psi|0)$

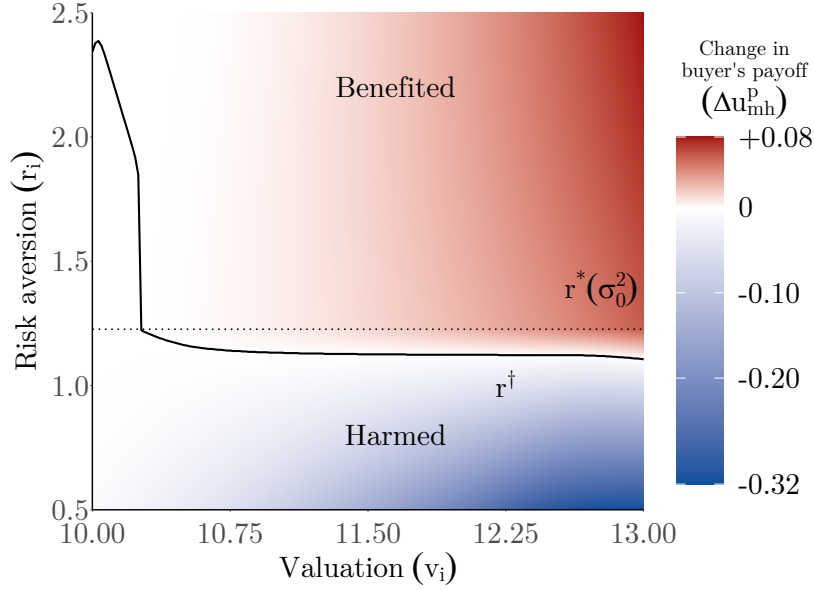
for some continuous $r^\dagger : [V^*(\underline{v}, \bar{r}|0), V^*(\bar{v}, \underline{r}|0)] \rightarrow (\underline{r}, r^\psi]$.³⁴

³³Aggregate buyer surplus decreases in expectation: $\mathbb{E}[M_{\text{mh}}^{\text{P}} - \tilde{b}^{\text{P}}(M_{\text{mh}}^{\text{P}}|1)] < \mathbb{E}[M_{\text{mh}}^* - \tilde{b}^*(M_{\text{mh}}^*|0)]$.

³⁴If $r^\psi = \bar{r}$, then $V^*(v_i, r_i|0) \leq V^*(\underline{v}, r^\psi|0)$ if and only if $(v_i, r_i) = (\underline{v}, \bar{r})$, so the set of unaffected buyers has measure zero.

As in section 3.1, the policy benefits the more risk-averse buyers while harming the most risk-tolerant ones. A major difference from section 3.1 is that some risk-averse buyers with low valuations are now harmed by the policy. This is because, as discussed above, the policy reduces the sensitivity of every buyer's effective valuation to his risk aversion by inducing maintenance. Indeed, when maintenance does not reduce the repair-cost risk (i.e., $\Delta_{\sigma^2} = 0$), a buyer i is harmed by the policy if and only if $r_i < r^\dagger(V^*(v_i, r_i|0))$, where $r^\dagger(V^*(v_i, r_i|0)) \leq r^\psi \leq r^*(\sigma_0^2)$. That is, when $\Delta_{\sigma^2} = 0$, no buyer who submits a contingent offer in the free market equilibrium is harmed by the policy.

Figure 7: Heterogeneous no-waiver policy effects on buyers' welfare under moral hazard



Note: The solid curve is the threshold of Proposition 14. Parameters as in Figure 6.

Equilibrium multiplicity. Given Proposition 10, the no-maintenance free market equilibrium is unique under any of the conditions of Lemma 3, each of which implies $\Gamma(1) < \Gamma(0)$. If, instead, $\Gamma(1) \geq \Gamma(0)$, it is possible that $\theta \in [\Gamma(0), \Gamma(1)]$. Then, by Proposition 10, the free market also has an equilibrium where the seller maintains. Relative to that equilibrium, the policy leaves maintenance unchanged, so section 3.1 provides the comparison between this free market equilibrium and the equilibrium under the policy.³⁵

In the case of equilibrium multiplicity, this section can also be seen as following a robust approach by focusing on the worst equilibrium. In the free market, realized total surplus is $\max_i V^*(v_i, r_i|0)$ in the equilibrium without maintenance and $\max_i V^*(v_i, r_i|1) - \theta$

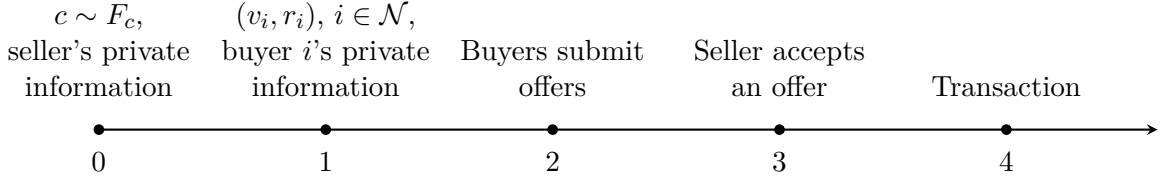
³⁵The policy can never deter maintenance when maintenance would take place in the free market equilibrium. To see this, suppose, to the contrary, that it does. By Propositions 10 and 11, this would require $\Gamma(1) \geq \theta > \Delta_\omega$, which contradicts Lemma 2.

in the equilibrium with maintenance, so maintenance is efficient if and only if $\theta < \mathbb{E}[\max_i V^*(v_i, r_i|1) - \max_i V^*(v_i, r_i|0)]$. Therefore, in the free market, the equilibrium without maintenance is the worst one (in terms of total surplus) if $\theta < \mathbb{E}[\max_i V^*(v_i, r_i|1) - \max_i V^*(v_i, r_i|0)]$.³⁶

3.3 Asymmetric information: inspections and adverse selection

Figure 8 presents the timeline of the game. The game is the same as the baseline one, except that the seller privately observes the repair cost c before accepting an offer. To simplify notation, we again drop information sets from agents' payoffs, bearing in mind that for every buyer i , at the information set $\mathcal{I}_i$ where his offer has been accepted, he knows (v_i, r_i) , and his beliefs over c are derived from Bayes' rule given the prior, the buyers' strategies, and the seller's strategy. At the information set $\mathcal{I}_S$ where the seller accepts an offer, she knows c .

Figure 8: Game with adverse selection: timeline



3.3.1 The free market equilibrium

The seller's payoff from accepting offer (b_i, d_i) is $\pi_S(b_i, d_i; r_S|c) = b_i - d_i\omega(c)$, where $\omega(c) := \kappa_S + \zeta c$. Therefore, when some offers are contingent and others are not, which offer wins depends on c , and winning is informative about c . For tractability, I characterize two kinds of free market equilibria: one in which every buyer waives inspections (Proposition 15) and one in which every buyer submits a contingent offer (Proposition 16). In such equilibria, winning the object is uninformative about c on the equilibrium path.

All-waive equilibrium. In the all-waive equilibrium, a buyer i who submits offer $(b_i, 0)$ and wins earns payoff $\pi_B(b_i, 0; v_i, r_i) = V(0, v_i, r_i) - b_i$. His effective valuation is $V(0, v_i, r_i)$. The equilibrium coincides with the free market one in the baseline setting, as characterized by Proposition 1 for $r^* > \bar{r}$.

³⁶For every (v, r) , $V^*(v, r|1) - V^*(v, r|0) \geq \Delta_\mu + \min\{r, (1 - \zeta)^2 r + \zeta^2 r_S\} \Delta_{\sigma^2}$, so $\theta < \Delta_\mu + \min\{\underline{r}, (1 - \zeta)^2 \underline{r} + \zeta^2 r_S\} \Delta_{\sigma^2}$ guarantees that the maintenance equilibrium is the efficient one in the free market. Also, given $\theta < \Delta_\omega$, $(\zeta^2 r_S - \underline{r}) \Delta_{\sigma^2} \leq (1 - \zeta) \Delta_\mu$ guarantees that the maintenance equilibrium is the efficient one in the free market.

Proposition 15. There is a symmetric monotone equilibrium in which every buyer waives inspections if and only if $\kappa \geq \bar{\kappa}_{\text{as}}^* := \mu + \bar{r}\sigma^2$. The buyer with the highest $V(0, v_i, r_i)$ wins the object.

Strikingly, in equilibrium, every buyer waives inspections if and only if the inspection cost κ is excessively high: higher than every buyer's disutility from bearing all the repair-cost risk on his own. That is, the inspection cost must be higher than the sum of the expected repair cost μ and the most risk-averse buyer's disutility $\bar{r}\sigma^2$ from bearing the repair-cost risk.

Here is the intuition behind this result. In equilibrium, buyer i with type $(\underline{v}, \bar{r})$ submits offer $(V(0, \underline{v}, \bar{r}), 0)$, where $V(0, \underline{v}, \bar{r}) = \underline{v} - \mu - \bar{r}\sigma^2$, and never wins the object. By deviating and submitting contingent offer $(V(0, \underline{v}, \bar{r}) + \kappa_S + \varepsilon, 1)$ instead, he more than compensates the seller for the inspection cost κ_S she will incur if she accepts the offer. Of course, by accepting a contingent offer, the seller also has to pay a share ζ of the repair cost c . Knowing c , the seller accepts the contingent offer only if $c \leq \varepsilon/\zeta$. Therefore, when c is low enough and every other buyer's offer is also low—which happens with positive probability—the seller accepts the deviating buyer's contingent offer. Also, the repair-cost risk vanishes (i.e., the mean and variance of c conditional on his offer being accepted vanish) as $\varepsilon \rightarrow 0$. Thus, if the buyer deviates to the contingent offer, his payoff from winning (which now happens with positive probability) is

$$\underline{v} - \kappa_B - \underbrace{(V(0, \underline{v}, \bar{r}) + \kappa_S + \varepsilon)}_{\text{bid}} - \underbrace{[(1 - \zeta)\mathbb{E}[c|\mathcal{I}_i] + (1 - \zeta)^2\bar{r}\text{Var}(c|\mathcal{I}_i)]}_{O(\varepsilon)} = \mu + \bar{r}\sigma^2 - \kappa - O(\varepsilon).$$

Due to favorable selection, the buyer gains from a contingent offer by completely avoiding the disutility $\mu + \bar{r}\sigma^2$ from the risk associated with a non-contingent offer. Therefore, for the deviation not to be profitable, the total inspection cost κ must be at least as large as that gain.

Corollary 15.1 compares the threshold $\bar{\kappa}_{\text{as}}^*$ with the corresponding threshold (for there to be no inspections in equilibrium) under symmetric information (i.e., as in section 3.1). It reveals that, due to adverse selection, buyers have remarkably stronger incentives to inspect when the seller has private information on c .

Corollary 15.1. Under symmetric information, every buyer waives inspections in equilibrium if and only if $\kappa \geq \kappa^* := \zeta(2 - \zeta)\bar{r}\sigma^2 - \zeta^2 r_S \sigma^2$. The threshold is lower than when the seller privately observes c : $\bar{\kappa}_{\text{as}}^* - \kappa^* = \mu + (1 - \zeta)^2 \bar{r}\sigma^2 + \zeta^2 r_S \sigma^2 > 0$.

All-contingent equilibrium. Let $\underline{\kappa}_{\text{as}}^* := \min_{r \in \{\underline{r}, \bar{r}\}} \min_{t \in [0, \bar{c}]} [(1 - \zeta)(\mathbb{E}[c | c \geq t] - \mu) + r(\text{Var}(c | c \geq t) - (1 - \zeta)^2 \sigma^2)]$, so that $\underline{\kappa}_{\text{as}}^* \leq \zeta(2 - \zeta)\underline{r}\sigma^2 < \bar{\kappa}_{\text{as}}^*$.³⁷ Proposition 16 provides

³⁷For $t = \bar{c}$, the expression inside the brackets is equal to $(1 - \zeta)(\bar{c} - \mu) - r(1 - \zeta)^2 \sigma^2$.

a sufficient condition for the all-contingent equilibrium to exist. The equilibrium coincides with the one under the no-waiver policy in the baseline setting (see Proposition 2).

Proposition 16. If $\kappa \leq \underline{\kappa}_{\text{as}}^*$, there is a symmetric monotone equilibrium in which every buyer submits a contingent offer. The buyer with the highest $V(1, v_i, r_i)$ wins the object.

A buyer who considers deviating to a non-contingent offer will face adverse selection.

The free market equilibrium objects. In the all-waive equilibrium, a buyer i who submits offer $(b_i, 0)$ and wins earns payoff $\pi_B(b_i, 0; v_i, r_i) = V(0, v_i, r_i) - b_i$. His effective valuation is $V(0, v_i, r_i)$. In the all-contingent equilibrium, a buyer i who submits offer $(b_i, 1)$ and wins earns payoff $\pi_B(b_i, 1; v_i, r_i) = V(1, v_i, r_i) - \tilde{b}(b_i, 1)$. His effective valuation is $V(1, v_i, r_i)$. Denote by

$$F_{V,\text{as}}^*(s|\kappa) := \begin{cases} \int_{\underline{r}}^{\bar{r}} \int_{\underline{v}}^{\bar{v}} \mathbf{I}\{V(1, v, r) \leq s\} f_{vr}(v, r) dv dr & \text{if } \kappa \leq \underline{\kappa}_{\text{as}}^*, \\ \int_{\underline{r}}^{\bar{r}} \int_{\underline{v}}^{\bar{v}} \mathbf{I}\{V(0, v, r) \leq s\} f_{vr}(v, r) dv dr & \text{if } \kappa \geq \bar{\kappa}_{\text{as}}^* \end{cases}$$

the CDF of a buyer's effective valuation in the free market equilibrium. The equilibrium effective bid is

$$\tilde{b}_{\text{as}}^*(s|\kappa) := \begin{cases} s - \int_{V(1, \underline{v}, \bar{r})}^s F_{V,\text{as}}^*(t|\kappa)^{N-1} dt / F_{V,\text{as}}^*(s|\kappa)^{N-1} & \text{if } \kappa \leq \underline{\kappa}_{\text{as}}^*, \\ s - \int_{V(0, \underline{v}, \bar{r})}^s F_{V,\text{as}}^*(t|\kappa)^{N-1} dt / F_{V,\text{as}}^*(s|\kappa)^{N-1} & \text{if } \kappa \geq \bar{\kappa}_{\text{as}}^*. \end{cases}$$

Denote by

$$M_{\text{as}}^*(\kappa) := \begin{cases} \max_i V(1, v_i, r_i) & \text{if } \kappa \leq \underline{\kappa}_{\text{as}}^*, \\ \max_i V(0, v_i, r_i) & \text{if } \kappa \geq \bar{\kappa}_{\text{as}}^* \end{cases}$$

the winning buyer's effective valuation. The equilibrium realized total surplus and the seller's equilibrium realized surplus are $M_{\text{as}}^*(\kappa) + \omega - \omega(c)$ and $\tilde{b}_{\text{as}}^*(M_{\text{as}}^*(\kappa)|\kappa) + \omega - \omega(c)$, respectively, if $\kappa \leq \underline{\kappa}_{\text{as}}^*$. The equilibrium realized total surplus and the seller's equilibrium realized surplus are $M_{\text{as}}^*(\kappa)$ and $\tilde{b}_{\text{as}}^*(M_{\text{as}}^*(\kappa)|\kappa)$, respectively, if $\kappa \geq \bar{\kappa}_{\text{as}}^*$. Buyer i 's interim expected surplus is $u_{\text{as}}^*(v_i, r_i|\kappa) := \int_{V(1, \underline{v}, \bar{r})}^{V(1, v_i, r_i)} F_{V,\text{as}}^*(t|\kappa)^{N-1} dt$ if $\kappa \leq \underline{\kappa}_{\text{as}}^*$, and $u_{\text{as}}^*(v_i, r_i|\kappa) := \int_{V(0, \underline{v}, \bar{r})}^{V(0, v_i, r_i)} F_{V,\text{as}}^*(t|\kappa)^{N-1} dt$ if $\kappa \geq \bar{\kappa}_{\text{as}}^*$. Below, κ is dropped from the arguments of $\tilde{b}_{\text{as}}^*$, M_{as}^* and u_{as}^* .

3.3.2 Equilibrium under the no-waiver policy

Assumption 2 is maintained, so that there is an equilibrium where every buyer inspects. A buyer i who submits offer b_i and wins earns payoff $\pi_B(b_i, 1; v_i, r_i) = V(1, v_i, r_i) - \tilde{b}(b_i, 1)$, and his effective valuation is $V(1, v_i, r_i)$, with CDF $F_V^p(s) \equiv \int_{\underline{r}}^{\bar{r}} \int_{\underline{v}}^{\bar{v}} \mathbf{I}\{V(1, v, r) \leq s\} f_{vr}(v, r) dv dr$, as in section 3.1. The seller's payoff from accepting buyer i 's offer is

$\pi_S(b_i, 1; r_S | c) = b_i - \omega(c)$. Proposition 17 describes the equilibrium. The equilibrium coincides with the one under the no-waiver policy in the baseline setting (see Proposition 2).

Proposition 17. Under the no-waiver policy, there is a symmetric monotone equilibrium: Each buyer i submits price $b^p(v_i, r_i) = \tilde{b}_{as}^p(V(1, v_i, r_i)) + \omega$ and, if he wins, he has an inspection performed, where $\tilde{b}_{as}^p(s) \equiv s - \int_{V(1, \underline{v}, \bar{r})}^s F_V^p(t)^{N-1} dt / F_V^p(s)^{N-1}$. The buyer with the highest $V(1, v_i, r_i)$ wins the object.³⁸

The winning buyer's effective valuation is $M_{as}^p \equiv \max_i V(1, v_i, r_i)$. The seller's equilibrium realized surplus is $\tilde{b}_{as}^p(M_{as}^p) + \omega - \omega(c)$. The winning buyer's equilibrium realized surplus is $M_{as}^p - \tilde{b}_{as}^p(M_{as}^p)$. $\tilde{M}_{as}^p := M_{as}^p + \omega - \omega(c)$ is the equilibrium realized total surplus. Buyer i 's interim expected surplus is $u_{as}^p(v_i, r_i) \equiv \int_{V(1, \underline{v}, \bar{r})}^{V(1, v_i, r_i)} F_V^p(t)^{N-1} dt$. M_{as}^p , $\tilde{b}_{as}^p$ and u_{as}^p coincide with M^p , $\tilde{b}^p$ and u^p of section 3.1. These are the realized surpluses of the equilibrium of Proposition 16.

3.3.3 No-waiver policy effects

Assume throughout this section that $\kappa \geq \bar{\kappa}_{as}^*$, so that the policy has an effect. Given assumption 2, for $\kappa \geq \bar{\kappa}_{as}^*$ to hold, it is necessary that $\kappa_S \geq (1 - \zeta)\mu + [\bar{r} - \zeta(2 - \zeta)r]\sigma^2 > 0$. This section compares the free market equilibrium where every buyer waives inspections with the equilibrium under the policy where every buyer inspects.

Effect on total and seller surplus. Proposition 18 shows that the policy causes an FOSD-decrease in total surplus and seller surplus.

Proposition 18. The policy FOSD-decreases

(i) total surplus: For every realization $(v_i, r_i)_{i \in \mathcal{N}}$ of the buyers' types and every c , $\tilde{M}_{as}^p < M_{as}^*$; and

(ii) seller surplus: For every x , $\Pr(\tilde{b}_{as}^p(M_{as}^p) + \omega - \omega(c) \leq x) \geq \Pr(\tilde{b}_{as}^*(M_{as}^*) \leq x)$.³⁹

By Corollary 15.1, given $\kappa \geq \bar{\kappa}_{as}^*$, it holds that $r^* > \bar{r}$ in the baseline setting of section 3.1. Therefore, the free market equilibrium under adverse selection is the same as in the baseline setting (see Proposition 1). The equilibrium under the policy is also the same as

³⁸It is the unique symmetric monotone equilibrium in which the seller accepts the highest price at every c . A seller acceptance rule that depends on c could potentially be sustained in equilibrium if it induced some buyer types not to inspect after winning (because in their beliefs over c conditional on winning, there is less uncertainty over c than in their prior beliefs).

³⁹ $\tilde{M}_{as}^p \leq M_{as}^* - \kappa + \zeta(2 - \zeta)\bar{r}\sigma^2 + \zeta(\mu - c) \leq M_{as}^* - (1 - \zeta)\mu - \zeta c$. The inequality in (ii) is strict if and only if $x \in (V(1, \underline{v}, \bar{r}) + \omega - \omega(\bar{c}), \tilde{b}_{as}^*(V(0, \bar{v}, \underline{r})))$. In expectation, $\Delta W_{as}^p := \mathbb{E}[\tilde{M}_{as}^p] - \mathbb{E}[M_{as}^*]$ and $\Delta \Pi_{as}^p := \mathbb{E}[\tilde{b}_{as}^p(M_{as}^p) + \omega - \omega(c)] - \mathbb{E}[\tilde{b}_{as}^*(M_{as}^*)]$ satisfy $\max\{\Delta W_{as}^p, \Delta \Pi_{as}^p\} \leq -\kappa + \zeta(2 - \zeta)\bar{r}\sigma^2 \leq -\mu$.

the one in the baseline setting (see Proposition 2). Hence, Proposition 18 compares the same two equilibria that Proposition 3 compares.

The difference is that, due to the seller's private information, the realized total surplus and seller surplus under the policy differ from their values in section 3.1 by $\omega - \omega(c) = \zeta(\mu - c) + \zeta^2 r_S \sigma^2$. The first term of $\omega - \omega(c)$ captures the fact that the seller does not price a contingency at prior beliefs over c but rather at the realized c . The second term captures the fact that the seller no longer bears risk by accepting a contingent offer.⁴⁰ The effect of the policy on total surplus can be decomposed as follows:

$$\begin{aligned} \widetilde{M}_{as}^p - M_{as}^* &= \overbrace{M_{as}^p - M_{as}^*}^{\text{<0; symmetric information effect}} + \overbrace{\omega - \omega(c)}^{\text{adjustment due to seller private information}} \leq \max_i [V(1, v_i, r_i) - V(0, v_i, r_i)] + \omega - \omega(c) \\ &\leq -\kappa + \zeta(2 - \zeta)\bar{r}\sigma^2 - \zeta^2 r_S \sigma^2 + \zeta(\mu - c) + \zeta^2 r_S \sigma^2 < 0, \end{aligned}$$

since $\kappa \geq \bar{\kappa}_{as}^* \equiv \mu + \bar{r}\sigma^2$.

Effect on aggregate buyer surplus. As we have already seen, $\kappa \geq \bar{\kappa}_{as}^*$ implies $r^* > \bar{r}$. Thus, the distribution of aggregate buyer surplus in the free market equilibrium under adverse selection is the same as in the baseline setting. Similarly, the distribution of aggregate buyer surplus in the no-waiver policy equilibrium under adverse selection is the same as in the baseline setting. Therefore, under the same conditions as in Proposition 4, the policy reduces aggregate buyer surplus.

Distributional effect on buyer surplus. The effect of the policy on buyer i 's interim expected surplus is as in section 3.1, and Proposition 5 applies with $r^\varphi = \bar{r}$. The policy harms the most risk-tolerant buyers and benefits the most risk-averse ones.

4 Conclusion

In this paper, I study the practice of homebuyers waiving inspections and recent *no-waiver* regulation aimed at preventing them from doing so. I show that the prevalence of inspection waivers in a seller's market can be explained by limited buyer bargaining power to negotiate seller remediation after an inspection. Even so, the desirability of a no-waiver regulation depends on the welfare criterion and on the details of the real estate landscape. When inspections are mainly a tool for risk-sharing between the buyer and the seller, the regulation decreases total, seller, and—under realistic assumptions—buyer welfare. This is because it distorts risk-sharing by inducing risk-tolerant buyers to shift—through inspection contingencies—part of the risk to the seller even though they are better

⁴⁰In expectation, total surplus in the no-waiver policy equilibrium is higher than in the baseline setting by $\zeta^2 r_S \sigma^2$.

equipped to bear it themselves. However, to the extent that risk-averse buyers tend to be less wealthy than both the sellers and the more risk-tolerant buyers, the regulation is beneficial under (sufficiently) inequality-averse social welfare criteria, since it benefits the more risk-averse buyers. The policy favors, for example, low-income homebuyers over wealthier ones, and individual homebuyers over corporate ones and real estate developers. Strikingly, under an aggregate buyer welfare criterion, a *ban* of inspections is beneficial.

The answer changes if homeowners maintain their homes inefficiently because they fail to internalize the positive externalities on future owners. The regulation can increase total and seller welfare by inducing homeowners to maintain their homes more efficiently. The regulation still decreases aggregate buyer welfare but benefits the less wealthy, more risk-averse buyers. Last, the regulation’s effectiveness in reducing information asymmetries between the seller and potential buyers seems limited. That is because adverse selection (i.e., the seller accepting offers with waived inspections when the house is in bad condition) already gives buyers very strong incentives to include inspection contingencies.

The analysis yields the following testable predictions. First, the share of offers that waive inspections is single-dipped in the buyer’s post-inspection bargaining power to negotiate seller remediation, as measured by the share of repair costs that the seller covers after an inspection. Second, this relationship is predicted to vary with the seller’s risk aversion. For sellers who are closer to risk-neutral (e.g., banks and real estate developers), the share of offers with waived inspections is expected to decrease monotonically with the buyer’s post-inspection bargaining power. By contrast, for more risk-averse sellers such as individual homeowners, the non-monotone relationship described above is predicted to hold. Third, buyers are more likely to waive inspections when the seller is more risk-averse. Fourth, the premium a contingent offer must pay (in the form of a higher price than offers with waived inspections) is expected to increase with the buyer’s post-inspection bargaining power and with the seller’s risk aversion. This is consistent with Reher and Valkanov’s (2024) finding that sellers with lower wealth (and thus probably lower risk-bearing capacity) require a higher premium for bearing the transaction risk associated with mortgage-financed offers (compared to all-cash offers). Fifth, to the extent that less wealthy buyers are more risk-averse, a no-waiver regulation should increase the share of houses bought by less wealthy buyers (e.g., FHA-financed borrowers). This prediction can be tested in Massachusetts, where the regulation has already taken effect.

Future work can use the framework developed here to study due diligence beyond real estate transactions. Consistent with my results, venture capitalists perform less due diligence in markets with intense investor competition, and deals with less due diligence have more dispersed returns but perform the same on average (Fu and Taylor, 2025). Relatedly, in Griffith’s (2020) survey, respondents attributed the purchase of representation and warranty (R&W) insurance to the seller’s bargaining power in mergers and acquisitions. My results can also explain why the purchase of R&W insurance is

more likely when acquisition targets operate in industries with higher target valuation uncertainty (Even-Tov et al., 2024). Last, future work can extend the model developed here by combining it with a model of post-inspection renegotiation or by examining the role of a third party who can bear the repair cost risk after a real estate transaction.⁴¹

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⁴¹Insurance that covers repair costs for issues that a home inspection missed has been offered (Lerner, 2021).

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A Appendix

A.1 Some useful results

Lemma 4 (Effective valuations). Let c be continuous and non-decreasing on $[\underline{r}, \bar{r}]$. The CDF of $v_i - c(r_i)$ is continuous and increasing on $[\underline{v} - c(\bar{r}), \bar{v} - c(\underline{r})]$, with density $\int_{\underline{r}}^{\bar{r}} f_{vr}(t + c(r), r) \mathbf{I}\{\underline{v} \leq t + c(r) \leq \bar{v}\} dr$ at t , positive on the interior of that interval, and the quantile is an increasing bijection of $(0, 1)$ onto that interior.

Every regime in the paper is a symmetric first-price auction in an effective valuation $v_i - c(r_i)$ with a continuous non-decreasing cost side c .

Lemma 5. Under assumption 1, for every type (v_i, r_i) , every contract $d \in \{0, 1\}$ and every realized repair cost $c \in [0, \bar{c}]$, $V(d, v_i, r_i) > 0$ and $v_i - \kappa - (1 - \zeta)\mu - \zeta c - (1 - \zeta)^2 r_i \sigma^2 > 0$.

$V(d, v_i, r_i) > 0$ and $v_i - \kappa - (1 - \zeta)\mu - \zeta c - (1 - \zeta)^2 r_i \sigma^2 > 0$ are the surpluses a sale generates (i) when both the buyer's and the seller's beliefs over c are given by the prior, and (ii) when the seller knows c but the buyer's beliefs over c are given by the prior, respectively. These are the cases arising in sections 3.1, 3.2 and 3.3.

Lemma 6. Let $N \geq 2$ values be drawn independently from a continuous, increasing CDF F on $[\underline{s}, \bar{s}]$, and let $\tilde{b}(s) := s - F(s)^{-(N-1)} \int_{\underline{s}}^s F(t)^{N-1} dt$ with $\tilde{b}(\underline{s}) := \underline{s}$. Then (i) at every rank $q \in (0, 1]$, $\tilde{b}(F^{-1}(q)) = \int_0^q F^{-1}(w) \frac{(N-1)w^{N-2}}{q^{N-1}} dw$; (ii) the expected winning bid equals the expected second-highest of the N values; (iii) $\tilde{b}$ is continuous and increasing on $[\underline{s}, \bar{s}]$; (iv) in every symmetric equilibrium where the bid is increasing in the value, the bid is $\tilde{b}$ on $(\underline{s}, \bar{s}]$; and (v) $\tilde{b}$ is a symmetric equilibrium bid.

A.2 Proofs

Proof of Lemma 4. Write, in this proof, F_V , f_V and Q_V for the CDF, the density claimed and the quantile of $v_i - c(r_i)$, and $\underline{s} := \underline{v} - c(\bar{r})$ and $\bar{s} := \bar{v} - c(\underline{r})$; every value of $v_i - c(r_i)$ lies in $[\underline{s}, \bar{s}]$, c being non-decreasing. We have

$$\begin{aligned} F_V(t) &= \int_{\underline{r}}^{\bar{r}} \int_{\underline{v}}^{\bar{v}} \mathbf{I}\{v \leq t + c(r)\} f_{vr}(v, r) dv dr \\ &= \int_{-\infty}^t \int_{\underline{r}}^{\bar{r}} f_{vr}(x + c(r), r) \mathbf{I}\{\underline{v} \leq x + c(r) \leq \bar{v}\} dr dx = \int_{-\infty}^t f_V(x) dx, \end{aligned}$$

where we have substituted $v = x + c(r)$ in the inner integral and exchanged the order of integration by Tonelli's theorem. Thus, F_V has density f_V and is therefore continuous. For $\underline{s} < t < \bar{s}$ the set $R_t := \{r \in [\underline{r}, \bar{r}] : \underline{v} < t + c(r) < \bar{v}\}$ is open in $[\underline{r}, \bar{r}]$, c being continuous, and non-empty: c takes every value between $c(\underline{r})$ and $c(\bar{r})$, and $\underline{v} - t < c(\bar{r})$, which is $t > \underline{s}$, together with $c(\underline{r}) < \bar{v} - t$, which is $t < \bar{s}$, puts a point of $[c(\underline{r}), c(\bar{r})]$ strictly inside

$(\underline{v} - t, \bar{v} - t)$. A non-empty set open in $[\underline{r}, \bar{r}]$ contains a non-degenerate interval, so R_t has positive length, and

$$f_V(t) \geq \int_{R_t} f_{vr}(t + c(r), r) dr > 0,$$

so F_V is increasing on $[\underline{s}, \bar{s}]$, its density being positive on $(\underline{s}, \bar{s})$, from $F_V(\underline{s}) = 0$ to $F_V(\bar{s}) = 1$, every value lying in $[\underline{s}, \bar{s}]$ and F_V being continuous, and its restriction there is a continuous increasing bijection onto $[0, 1]$. Its inverse Q_V is therefore continuous and increasing on $[0, 1]$ with $Q_V(0) = \underline{s}$ and $Q_V(1) = \bar{s}$, so Q_V maps $(0, 1)$ onto $(\underline{s}, \bar{s})$, which is the bijection claimed. **Q.E.D.**

Proof of Lemma 5. Each branch of $V(d, v_i, r_i)$ is increasing in v_i and decreasing in r_i , as is $v_i - \kappa - (1 - \zeta)\mu - \zeta c - (1 - \zeta)^2 r_i \sigma^2$, and the latter is decreasing in c ; so it suffices to sign the three quantities at $v_i = \underline{v}$, $r_i = \bar{r}$ and $c = \bar{c}$, where each is $\underline{v}$ less one term of the maximum in assumption 1: $V(0, \underline{v}, \bar{r}) = \underline{v} - \mu - \bar{r} \sigma^2 > 0$, $V(1, \underline{v}, \bar{r}) = \underline{v} - \kappa - \mu - (1 - \zeta)^2 \bar{r} \sigma^2 - \zeta^2 r_S \sigma^2 > 0$ and $\underline{v} - \kappa - (1 - \zeta)\mu - \zeta \bar{c} - (1 - \zeta)^2 \bar{r} \sigma^2 > 0$. **Q.E.D.**

Proof of Lemma 6. Write M and M_2 for the highest and the second-highest of the N values. Parts (i)–(iii) are computed in rank space: F is continuous and increasing, so $F(\cdot)$ maps each value to its rank, $F(F^{-1}(q)) = q$ and $F^{-1}(F(x)) = x$, and the N ranks are i.i.d. $U[0, 1]$.

Part (i). Fix $q \in (0, 1]$ and set $s = F^{-1}(q)$ in the definition of $\tilde{b}$. Integrating by parts, and then substituting $w = F(t)$, so that $t = F^{-1}(w)$ and w runs from 0 to q as t runs from $\underline{s}$ to $F^{-1}(q)$,

$$\begin{aligned} \int_{\underline{s}}^{F^{-1}(q)} F(t)^{N-1} dt &= \left[t F(t)^{N-1} \right]_{\underline{s}}^{F^{-1}(q)} - \int_{\underline{s}}^{F^{-1}(q)} t(N-1) F(t)^{N-2} dF(t) \\ &= q^{N-1} F^{-1}(q) - \int_0^q F^{-1}(w)(N-1) w^{N-2} dw, \end{aligned}$$

the bracket vanishing at $\underline{s}$ because $F(\underline{s}) = 0$. Hence

$$\tilde{b}(F^{-1}(q)) = F^{-1}(q) - \frac{1}{q^{N-1}} \int_{\underline{s}}^{F^{-1}(q)} F(t)^{N-1} dt = \int_0^q F^{-1}(w) \frac{(N-1)w^{N-2}}{q^{N-1}} dw,$$

where $(N-1)w^{N-2}/q^{N-1}$ is the density on $[0, q]$ of the largest of $N-1$ ranks all known to lie below q , so the bidder is offering the highest rival value he expects to face conditional on beating it.

Part (ii). The rank $F(M)$ is the largest of N i.i.d. $U[0, 1]$ draws, of density Nq^{N-1} , and $M = F^{-1}(F(M))$, so part (i) and Fubini's theorem give

$$\mathbb{E}[\tilde{b}(M)] = \int_0^1 \tilde{b}(F^{-1}(q)) Nq^{N-1} dq = N(N-1) \int_0^1 \int_0^q F^{-1}(w) w^{N-2} dw dq$$

$$= N(N-1) \int_0^1 F^{-1}(w) w^{N-2} (1-w) dw,$$

the factor q^{N-1} canceling in the first equality. The rank $F(M_2)$ is the second largest of the same N draws, of CDF $q^N + Nq^{N-1}(1-q)$ and hence density $N(N-1)q^{N-2}(1-q)$, so $M_2 = F^{-1}(F(M_2))$ has

$$\mathbb{E}[M_2] = \int_0^1 F^{-1}(w) N(N-1) w^{N-2} (1-w) dw,$$

the same integral. Hence $\mathbb{E}[\tilde{b}(M)] = \mathbb{E}[M_2]$.

Part (iii). Substituting $w = qz$ in part (i), with $dw = qdz$, so that the weight $(N-1)w^{N-2}dw/q^{N-1}$ becomes $(N-1)z^{N-2}dz$, at every $q \in (0, 1]$,

$$\tilde{b}(F^{-1}(q)) = \int_0^1 F^{-1}(qz) (N-1) z^{N-2} dz,$$

where F^{-1} is continuous and increasing on $(0, 1]$ with $F^{-1}(q) \rightarrow \underline{s}$ as $q \rightarrow 0$, F being continuous and increasing on $[\underline{s}, \bar{s}]$ with $F(\underline{s}) = 0$. At $q < q'$, $F^{-1}(qz) < F^{-1}(q'z)$ and $(N-1)z^{N-2} > 0$ at every $z \in (0, 1]$, so $\tilde{b}(F^{-1}(q)) < \tilde{b}(F^{-1}(q'))$. The integrand is bounded by $(N-1) \max\{|\underline{s}|, |\bar{s}|\}$ and continuous in q at every $z \in (0, 1]$, so $q \mapsto \tilde{b}(F^{-1}(q))$ is continuous by dominated convergence, and, $F^{-1}(qz) \rightarrow \underline{s}$ at every such z , tends to $\int_0^1 \underline{s} (N-1) z^{N-2} dz = \underline{s}$ as $q \rightarrow 0$. Since $\tilde{b}(s) = \tilde{b}(F^{-1}(F(s)))$ at every $s \in (\underline{s}, \bar{s}]$, with F continuous and increasing there and $F(s) \rightarrow 0$ as $s \rightarrow \underline{s}$, $\tilde{b}$ is continuous and increasing on $(\underline{s}, \bar{s}]$, as the composition of two continuous increasing maps, and $\tilde{b}(s) \rightarrow \underline{s}$ as $s \rightarrow \underline{s}$. Extended by $\tilde{b}(\underline{s}) := \underline{s}$, it is therefore continuous and increasing on $[\underline{s}, \bar{s}]$.

Part (iv). Let b be the bid in a symmetric equilibrium in which b is increasing, and $u(s) := F(s)^{N-1}[s - b(s)]$ the interim surplus of a bidder with value s , who wins exactly when his value is the largest, b being increasing and ties having probability zero, F being continuous. For $\underline{s} \leq s' < s \leq \bar{s}$,

$$\begin{aligned} u(s) &= F(s)^{N-1} [s - b(s)] \geq F(s')^{N-1} [s - b(s')], \\ \implies F(s')^{N-1} &\leq \frac{u(s) - u(s')}{s - s'} \leq F(s)^{N-1}, \\ \implies u'(s) &= F(s)^{N-1}, \\ \implies u(s) &= \int_{\underline{s}}^s F(t)^{N-1} dt, \\ \implies b(s) &= s - \frac{u(s)}{F(s)^{N-1}} = \tilde{b}(s), \end{aligned}$$

the first because bidding $b(s')$ is available to the bidder with value s and wins with probability $F(s')^{N-1}$; the second from the first at (s, s') and at (s', s) , divided by $s - s' > 0$; the third from $s' \uparrow s$ and $s \downarrow s'$ in the second, both bounds being continuous; the fourth

because the second bounds the difference quotients of u by 1, so u is Lipschitz and the integral of its derivative, and $u(\underline{s}) = F(\underline{s})^{N-1}[\underline{s} - b(\underline{s})] = 0$; the fifth by $F(s)^{N-1} > 0$ at $s > \underline{s}$ and the definition of u .

Part (v). Let the other $N - 1$ bidders bid $\tilde{b}$ of their values, and write, in this part, $A(s) := \int_{\underline{s}}^s F(t)^{N-1} dt$, so that $F(s)^{N-1}[s - \tilde{b}(s)] = A(s)$ by the definition of $\tilde{b}$, and A is convex, its derivative F^{N-1} being non-decreasing. By part (iii), $\tilde{b}$ is continuous and increasing on $[\underline{s}, \bar{s}]$, so every bid in $[\tilde{b}(\underline{s}), \tilde{b}(\bar{s})]$ is $\tilde{b}(s')$ for some $s' \in [\underline{s}, \bar{s}]$, ties have probability zero, and a bid $\tilde{b}(s')$, $s' \in [\underline{s}, \bar{s}]$, wins with probability $F(s')^{N-1}$. A bidder with value s who bids $\tilde{b}(s')$ therefore earns

$$F(s')^{N-1} [s - \tilde{b}(s')] = A(s') + F(s')^{N-1}(s - s') \leq A(s),$$

the inequality by the convexity of A , with $A'(s') = F(s')^{N-1}$. A bid below $\tilde{b}(\underline{s})$ wins with probability zero and earns $0 = A(\underline{s}) \leq A(s)$, and a bid above $\tilde{b}(\bar{s})$ wins with probability one at a higher price than $\tilde{b}(\bar{s})$, which the display covers at $s' = \bar{s}$. Bidding $\tilde{b}(s)$ earns $A(s)$, so it is a best response, and $\tilde{b}$ is a symmetric equilibrium bid. **Q.E.D.**

Proof of Lemma 1. The seller accepts the largest effective bid, so whether buyer i wins depends on b_i and d_i only through $\tilde{b}(b_i, d_i) = b_i - d_i\omega$. Therefore, offers $(b_i, 0)$ and $(b_i + \omega, 1)$ win with the same probability for every b_i , and shift the buyer's payoff if he wins by $V(1, v_i, r_i) - V(0, v_i, r_i)$. $\Phi(r_i) := V(1, v_i, r_i) - V(0, v_i, r_i) = -\kappa + \zeta(2 - \zeta)r_i\sigma^2 - \zeta^2 r_S \sigma^2$ is independent of v_i and affine in r_i with slope $\zeta(2 - \zeta)\sigma^2 > 0$, crossing zero once, at $r^* \equiv [\kappa + \zeta^2 r_S \sigma^2] / [\zeta(2 - \zeta)\sigma^2]$. If $r_i > r^*$ and $d_i = 0$ and i wins with positive probability, $(b_i + \omega, 1)$ would raise buyer i 's payoff, so for $r_i > r^*$ it must be that $d_i = 1$ in equilibrium. Similarly, for $r_i < r^*$, $d_i = 0$ must hold in equilibrium. Last,

$$\frac{\partial r^*}{\partial \zeta} = -\frac{2(1 - \zeta)\kappa}{\zeta^2(2 - \zeta)^2\sigma^2} + \frac{2r_S}{(2 - \zeta)^2},$$

and the result follows. **Q.E.D.**

Proof of Proposition 1. The seller's payoff from accepting buyer i 's offer (b_i, d_i) is $\pi_S(b_i, d_i; r_S) = \tilde{b}(b_i, d_i) = b_i - d_i\omega$. Buyer i 's payoff when his offer is accepted is

$$V(d_i, v_i, r_i) - \tilde{b}(b_i, d_i) = \begin{cases} v_i - b_i - \mu - r_i\sigma^2 & \text{if } d_i = 0 \\ v_i - b_i - \kappa_B - (1 - \zeta)\mu - r_i(1 - \zeta)^2\sigma^2 & \text{if } d_i = 1. \end{cases}$$

By Lemma 1, $d^*(r_i) = \mathbf{I}\{r_i \geq r^*\}$, and the game is a symmetric first-price auction where each buyer chooses his effective bid with i.i.d. (effective) valuations $V^*(v_i, r_i)$. Define $m_0(r) := \mu + r\sigma^2$, $m_1(r) := \kappa + \mu + r(1 - \zeta)^2\sigma^2 + r_S\zeta^2\sigma^2$, and $m(r) := \min\{m_0(r), m_1(r)\}$ continuous and increasing, so that $V^*(v_i, r_i) = v_i - m(r_i)$. $V^*(v_i, r_i)$ has CDF $F_V^*(y) =$

$\Pr(v_i - m(r_i) \leq y) = \mathbb{E}[F_{v|r}(y + m(r_i)|r_i)]$, increasing on $[\underline{v} - m(\bar{r}), \bar{v} - m(\underline{r})]$.

It follows, by Lemma 4 and Lemma 6(iv)–(v), that in equilibrium, the effective bidding formula is

$$\tilde{b}^*(s) = s - \frac{\int_{\underline{s}}^s F_V^*(t)^{N-1} dt}{F_V^*(s)^{N-1}},$$

where $\underline{s} := V^*(\underline{v}, \bar{r}) = \underline{v} - m(\bar{r})$. No offer does better: By the proof of Lemma 1, at a given effective bid a buyer who wins earns at most $V^*(v_i, r_i)$ less that bid; and the seller accepts, every effective bid being at least $V^*(\underline{v}, \bar{r}) > 0$, by Lemma 6(iii) and Lemma 5. **Q.E.D.**

Proof of Proposition 2. The winner chooses d_i at fixed b_i , and the two branches of π_B at the prior moments differ by $-\kappa_B + \zeta\mu + \zeta(2 - \zeta)r_i\sigma^2$, free of v_i and b_i and increasing in r_i with a zero at r^n . The election rule is therefore $d^n(r_i) := \mathbf{I}\{r_i \geq r^n\}$, and winning at b_i pays $v_i - b_i - \min\{\mu + r_i\sigma^2, \kappa_B + (1 - \zeta)\mu + (1 - \zeta)^2r_i\sigma^2\}$. Dividing assumption 2 by $\zeta(2 - \zeta)\sigma^2 > 0$ gives $r^n \leq \underline{r}$, so $d^n(r_i) = 1$ at every type and that payoff is $V(1, v_i, r_i) - \tilde{b}(b_i, 1)$.

The no-waiver policy bars acceptance conditional on a waiver, so the seller ranks offers by price, which by the election rule is ranking by the effective bid; and every effective bid is at least $V(1, \underline{v}, \bar{r}) > 0$, by Lemma 6(iii) and Lemma 5, so selling dominates retaining. The game is a symmetric first-price auction in the i.i.d. $V(1, v_i, r_i)$, whose cost side $\kappa + \mu + (1 - \zeta)^2r\sigma^2 + \zeta^2\sigma^2r_S$ is continuous and increasing in r , and the proof of Proposition 1 applies with m replaced by it: Each buyer bids $\tilde{b}^P(V(1, v_i, r_i))$, and the price he submits is that bid plus ω .

At the winner the buyer earns $V(1, v_i, r_i) - \tilde{b}^P(V(1, v_i, r_i))$ and the seller $\tilde{b}^P(V(1, v_i, r_i))$, so the effective bid cancels in the sum: Realized total surplus is $V(1, v_i, r_i)$, which at the winner is M^P . **Q.E.D.**

Proof of Proposition 3. *Part (i): Realized total surplus.* Since $V^*(v_j, r_j) = \max_{d \in \{0,1\}} V(d, v_j, r_j)$ for every $j \in \mathcal{N}$ and every (v_j, r_j) , we get

$$M^* - M^P = \min_{j \in \mathcal{N}} [(M^* - V^*(v_j, r_j)) + (V^*(v_j, r_j) - V(1, v_j, r_j))] \geq 0,$$

where $M^* \geq V^*(v_j, r_j)$ and $V^*(v_j, r_j) \geq V(1, v_j, r_j)$ for every j and (v_j, r_j) . Thus, $M^* = M^P$ if and only if for some j , $V^*(v_j, r_j) = M^*$ and $V^*(v_j, r_j) = V(1, v_j, r_j)$. By the proof of Lemma 1,

$$V(1, v_j, r_j) - V^*(v_j, r_j) = \begin{cases} 0 & r_j \geq r^*, \\ -\zeta(2 - \zeta)\sigma^2(r^* - r_j) & r_j < r^*. \end{cases}$$

Hence, $M^P < M^*$ if and only if $r_j < r^*$ for every j with $V^*(v_j, r_j) = M^*$.

Part (ii): Seller surplus. Both regimes are symmetric first-price auctions in a scalar effective valuation, of CDF F_V^* and F_V^P respectively, each continuous and increasing by Lemma 4, so Lemma 6 applies to each. Write $s^g(q)$ for the effective valuation at rank q in regime g , which is the inverse of $F_V^g(\cdot)$.

Step 1: As in the proof of Proposition 1, $V^*(v_j, r_j) = v_j - m(r_j)$ and $V(1, v_j, r_j) = v_j - m_1(r_j)$, with m and m_1 continuous and increasing with $m_1(r) - m(r) = \zeta(2 - \zeta)\sigma^2 \max\{r^* - r, 0\}$. At every t , $F_V^P(t) - F_V^*(t) = \Pr(v_j - m_1(r_j) \leq t) - \Pr(v_j - m(r_j) \leq t) = \Pr(v_j - m_1(r_j) \leq t < v_j - m(r_j)) \geq 0$. Since this holds at every t , $s^P(q) \leq s^*(q)$ at every rank $q \in (0, 1]$. The event $\{v_j - m_1(r_j) \leq t < v_j - m(r_j)\}$ requires $m_1(r_j) > m(r_j)$, that is $r_j < r^*$, and hence $r_j \leq \check{r}$, where $\check{r} := \min\{r^*, \bar{r}\}$. $\Pr(v_j - m_1(r_j) \leq t < v_j - m(r_j))$ is positive if and only if $t \in I := (\underline{v} - m_1(\check{r}), \bar{v} - m(\underline{r})) = (V(1, \underline{v}, \check{r}), V^*(\bar{v}, \underline{r}))$, which is non-empty because $\check{r} > \underline{r}$, $r^* > \underline{r}$ being maintained where Proposition 3 is stated, because m_1 is increasing, and because $m_1 \geq m$. Take the three positions of t in turn.

1. If $t \leq \underline{v} - m_1(\check{r})$, then $v_j - m_1(r_j) \leq t < v_j - m(r_j)$ requires $r_j \leq \check{r}$, so given that m_1 is increasing, $v_j - m_1(r_j) \geq \underline{v} - m_1(\check{r}) \geq t$, the first inequality an equality only when $v_j = \underline{v}$ and $r_j = \check{r}$, which happens with probability 0. Thus, $\Pr(v_j - m_1(r_j) \leq t < v_j - m(r_j)) = 0$.
2. If $t \geq \bar{v} - m(\underline{r})$, $v_j - m(r_j) \leq \bar{v} - m(\underline{r}) \leq t$, so $\Pr(v_j - m_1(r_j) \leq t < v_j - m(r_j)) = 0$.
3. Assume $t \in I$. Then, $v_j - m_1(r_j) \leq t < v_j - m(r_j)$ if and only if $r_j < r^*$ and $v_j \in (t + m(r_j), t + m_1(r_j)] \cap [\underline{v}, \bar{v}]$, which is a non-degenerate interval if and only if $m(r_j) < \bar{v} - t$, $m_1(r_j) > \underline{v} - t$, and $m(r_j) < m_1(r_j)$. Any $r_j \in [\underline{r}, r_c)$ with $r_c = \min\{\check{r}, r'_c\}$, where r'_c solves $m(r'_c) = \bar{v} - t$ if $m(\bar{r}) \geq \bar{v} - t$ and $r'_c = \bar{r}$ otherwise, satisfies $m(r_j) < m_1(r_j)$ (because $r_j < r^*$) and $m(r_j) < \bar{v} - t$. Both $\check{r}$ and r'_c exceed $\underline{r}$, the latter because $t < \bar{v} - m(\underline{r})$, so $r_c > \underline{r}$. Also, $m_1(r_c) > \underline{v} - t$. To see this, notice that if (a) $r_c = \check{r}$, then $m_1(r_c) = m_1(\check{r}) > \underline{v} - t$ because $t > \underline{v} - m_1(\check{r})$, while if (b) $m(r_c) = \bar{v} - t$, then $m_1(r_c) = \bar{v} - t + m_1(r_c) - m(r_c) > \underline{v} - t$ because $\bar{v} > \underline{v}$ and $m_1 \geq m$. Given that m_1 is continuous and $m_1(r_c) > \underline{v} - t$, there exists $r_b \in [\underline{r}, r_c)$ with $m_1(r_j) > \underline{v} - t$ at every $r_j \in (r_b, r_c)$, so that at every such r_j all three conditions hold and $(t + m(r_j), t + m_1(r_j)] \cap [\underline{v}, \bar{v}]$ is non-degenerate. We conclude that if $t \in I$, then there exist r_c and r_b with $r_c > r_b \geq \underline{r}$ such that

$$\Pr(v_j - m_1(r_j) \leq t < v_j - m(r_j)) \geq \int_{r_b}^{r_c} \int_{\max\{t+m(r), \underline{v}\}}^{\min\{t+m_1(r), \bar{v}\}} f_{vr}(v, r) dv dr > 0,$$

the inequality because every (v, r) in the region of integration meets the event, and its positivity because that region is non-degenerate at every $r \in (r_b, r_c)$ and f_{vr} is positive.

Step 2: For every rank $q \in (0, 1]$, Lemma 6(i) then gives

$$\begin{aligned} & \tilde{b}^*(s^*(q)) - \tilde{b}^p(s^p(q)) \\ &= \int_0^q [s^*(w) - s^p(w)] \frac{(N-1)w^{N-2}}{q^{N-1}} dw \geq 0, \end{aligned}$$

where $s^*(w) \geq s^p(w)$ by Step 1.

Step 3: In each regime g , seller surplus is the winner's effective bid, $\tilde{b}^g(\cdot)$ is continuous and increasing, and M^g is the largest of N independent draws from $F_V^g(\cdot)$, so at every x , $\Pr(\tilde{b}^g(M^g) \leq x) = F_V^g((\tilde{b}^g)^{-1}(x))^N$, the inverse bid read as the minimum of the support below the lowest bid and as the maximum of the support above the highest. $s^g(\cdot)$ is increasing too, so at every $q \in (0, 1]$,

$$\tilde{b}^g(s^g(q)) \leq x \iff q \leq F_V^g((\tilde{b}^g)^{-1}(x)). \quad (1)$$

Write $q^g(x) := F_V^g((\tilde{b}^g)^{-1}(x))$. At $q^*(x) = 0$ the two ranks are ordered outright; at $q^*(x) > 0$ we have

$$\begin{aligned} & \tilde{b}^p(s^p(q^*(x))) \leq \tilde{b}^*(s^*(q^*(x))) \leq x \implies q^*(x) \leq q^p(x) \implies \\ & \Pr(\tilde{b}^p(M^p) \leq x) \geq \Pr(\tilde{b}^*(M^*) \leq x) \end{aligned}$$

where the first inequality follows by Step 2, and where we have used (1).

Step 4: Write $\check{s} := V(1, \underline{v}, \check{r}) = \underline{v} - m_1(\check{r})$ and $w_0 := F_V^p(\check{s})$. By Step 1, $F_V^p(t) > F_V^*(t)$ for $t \in I \equiv (\check{s}, V^*(\bar{v}, \underline{r}))$ and $F_V^p(t) = F_V^*(t)$ for $t \notin I$. Therefore, $s^p(w) = s^*(w)$ for $w \in (0, w_0]$ and $s^p(w) < s^*(w)$ for $w \in (w_0, 1]$. Given Step 2, this implies that $\tilde{b}^*(s^*(q)) > \tilde{b}^p(s^p(q))$ if and only if $q > w_0$.

If $0 < q^p(x) \leq w_0$, then $\tilde{b}^p(s^p(q^p(x))) = \tilde{b}^*(s^*(q^p(x))) \leq x$, so $q^p(x) \leq q^*(x)$. If $w_0 < q^*(x) < 1$, then $\tilde{b}^p(s^p(q^*(x))) < \tilde{b}^*(s^*(q^*(x))) \leq x$, so $q^p(x) > q^*(x)$. Therefore, given also $\Pr(\tilde{b}^g(M^g) \leq x) = q^g(x)^N$, we get the following equivalence:

$$\begin{aligned} & \Pr(\tilde{b}^p(M^p) \leq x) > \Pr(\tilde{b}^*(M^*) \leq x) \iff q^*(x) < q^p(x) \iff \\ & w_0 < q^p(x) \text{ and } q^*(x) < 1 \iff \tilde{b}^p(\check{s}) < x < \tilde{b}^*(V^*(\bar{v}, \underline{r})). \end{aligned}$$

Q.E.D.

Proof of Proposition 4. Step 1 writes aggregate buyer surplus in each regime as a function of the winner's rank; Step 2 writes the rank gap $g := s^* - s^p$ as an integral of the distortion's posterior mean along a homotopy between the two cost sides; Steps 3 and 4 show that mean is non-decreasing in the effective valuation, so g is non-decreasing; Step 5 concludes for the no-waiver policy and Step 6 for the ban; Step 7 shows that a ban

FOSD-decreases total and seller surplus.

Step 1: Aggregate buyer surplus through the winner's rank. Recall from the proof of Proposition 3 $s^g(q)$, the effective valuation at rank q in regime $g \in \{*, p\}$, the inverse of F_V^g , each F_V^g being continuous and increasing on its support by Lemma 4. The ranks $F_V^*(V^*(v_i, r_i))$, $i \in \mathcal{N}$, are therefore i.i.d. $U[0, 1]$, and so are the ranks $F_V^p(V(1, v_i, r_i))$; with $q_{(N)}$ the largest of N i.i.d. $U[0, 1]$ ranks, of density Nq^{N-1} and in $(0, 1)$ almost surely, M^g has the law of $s^g(q_{(N)})$. Write, in this proof, $k_q(w) := (N-1)w^{N-2}/q^{N-1}$ for $0 \leq w \leq q$, with $\int_0^q k_q(w)dw = 1$, and $R^g(q) := s^g(q) - \tilde{b}^g(s^g(q))$ for the winner's surplus at rank $q \in (0, 1]$, so that $M^g - \tilde{b}^g(M^g)$ has the law of $R^g(q_{(N)})$. By Lemma 6(i), with $g(q) := s^*(q) - s^p(q)$,

$$R^g(q) = \int_0^q [s^g(q) - s^g(w)] k_q(w)dw, \quad R^p(q) - R^*(q) = \int_0^q [g(w) - g(q)] k_q(w)dw.$$

Step 2: The rank gap as an integral of the distortion's posterior mean. Recall m , m_0 and m_1 from the proof of Proposition 1 and the distortion $\varphi(r) = m_1(r) - m(r)$ from section 3.1. Given $\varepsilon \in (0, [\zeta(2 - \zeta)]^{-1} - 1)$, for every $\alpha \in (-\varepsilon, 1 + \varepsilon)$ define $c_\alpha(r) := m(r) + \alpha\varphi(r) = (1 - \alpha)m(r) + \alpha m_1(r)$ and $V_\alpha(v_i, r_i) := v_i - c_\alpha(r_i)$, with CDF F_α , density f_α and quantile $Q_\alpha(q)$ at $q \in (0, 1)$, so that $c_0 = m$, $c_1 = m_1$, $Q_0 = s^*$ and $Q_1 = s^p$. c_α is continuous and increasing in r for every such α : $m = m_0$ on $[\underline{r}, r^*]$ and $m = m_1$ on $[r^*, \bar{r}]$ by the proof of Lemma 1, so $c'_\alpha(r) = \sigma^2[1 - \alpha\zeta(2 - \zeta)] > 0$ for $r < r^*$ and $c'_\alpha(r) = (1 - \zeta)^2\sigma^2 > 0$ for $r > r^*$. Write $\bar{f}(x, r) := f_{vr}(x, r)\mathbf{I}\{\underline{v} \leq x \leq \bar{v}\}$ for the joint density of (v_i, r_i) extended by zero, so that $\bar{f}(t + c_\alpha(r), r)$ is the joint density of $(V_\alpha(v_i, r_i), r_i)$ at (t, r) . By Lemma 4, F_α is continuous and increasing on $[\underline{v} - c_\alpha(\bar{r}), \bar{v} - c_\alpha(\underline{r})]$ with density $f_\alpha(t) = \int_{\underline{r}}^{\bar{r}} \bar{f}(t + c_\alpha(r), r)dr$, positive on the interior, and Q_α is continuous and increasing on $(0, 1)$. Write $h_\alpha(t) := \int_{\underline{r}}^{\bar{r}} \varphi(r) \bar{f}(t + c_\alpha(r), r)dr / f_\alpha(t)$ on that interior, the posterior mean $\mathbb{E}[\varphi(r_i) | V_\alpha(v_i, r_i) = t]$ of the distortion given the effective valuation. Since $F_\alpha(t) = \int_{\underline{r}}^{\bar{r}} \int_{-\infty}^{t+c_\alpha(r)} \bar{f}(x, r)dxdr$ with $\partial c_\alpha(r)/\partial \alpha = \varphi(r)$, $\bar{f}$ is bounded, and at every (t, α) the inner integral is differentiable in α except at the finitely many r with $t + c_\alpha(r) \in \{\underline{v}, \bar{v}\}$, $f_{vr}(\cdot, r)$ being continuous, dominated convergence gives $\partial F_\alpha(t)/\partial \alpha = \int_{\underline{r}}^{\bar{r}} \varphi(r) \bar{f}(t + c_\alpha(r), r)dr$, equal to $f_\alpha(t)h_\alpha(t)$ on the interior, and both $\partial F_\alpha/\partial \alpha$ and $\partial F_\alpha/\partial t = f_\alpha$ are continuous in (t, α) , their integrands being bounded and continuous in (t, α) at every r outside such a finite set. At $q \in (0, 1)$, $f_\alpha(Q_\alpha(q)) > 0$, so the implicit function theorem at $F_\alpha(Q_\alpha(q)) = q$ and the fundamental theorem of calculus give

$$\frac{\partial Q_\alpha(q)}{\partial \alpha} = -\frac{f_\alpha(Q_\alpha(q)) h_\alpha(Q_\alpha(q))}{f_\alpha(Q_\alpha(q))} = -h_\alpha(Q_\alpha(q)),$$

$$g(q) = Q_0(q) - Q_1(q) = \int_0^1 h_\alpha(Q_\alpha(q)) d\alpha.$$

Step 3: Log-submodularity of $\bar{f}(t + c_\alpha(r), r)$. Fix $\alpha \in [0, 1]$, $t, t' \in (\underline{v} - c_\alpha(\bar{r}), \bar{v} - c_\alpha(\underline{r}))$

with $t < t'$, and $r, r' \in [\underline{r}, \bar{r}]$ with $r < r'$. We want to show that $\bar{f}(t + c_\alpha(r), r)$ is log-submodular, that is,

$$\bar{f}(t' + c_\alpha(r'), r') \bar{f}(t + c_\alpha(r), r) \leq \bar{f}(t' + c_\alpha(r), r) \bar{f}(t + c_\alpha(r'), r'),$$

which automatically holds when the left-hand side is zero. When it is positive, $t + c_\alpha(r) \in [\underline{v}, \bar{v}]$ and $t' + c_\alpha(r') \in [\underline{v}, \bar{v}]$, so given that c_α is increasing, $t' + c_\alpha(r) \in [\underline{v}, \bar{v}]$ and $t + c_\alpha(r') \in [\underline{v}, \bar{v}]$ as well. Therefore, $\bar{f} = f_{vr}$ for all four arguments of $\bar{f}$. We then have

$$\frac{f_{vr}(t' + c_\alpha(r'), r')}{f_{vr}(t + c_\alpha(r'), r')} \leq \frac{f_{vr}(t' + c_\alpha(r), r')}{f_{vr}(t + c_\alpha(r), r')} \leq \frac{f_{vr}(t' + c_\alpha(r), r)}{f_{vr}(t + c_\alpha(r), r)}.$$

The first inequality holds because by log-concavity of $f_{v|r}(\cdot|r') = f_{vr}(\cdot, r')/f_r(r')$,

$$\begin{aligned} f_{vr}(t' + c_\alpha(r), r') &\geq f_{vr}(t + c_\alpha(r), r')^{1-\lambda} f_{vr}(t' + c_\alpha(r'), r')^\lambda, \text{ and} \\ f_{vr}(t + c_\alpha(r'), r') &\geq f_{vr}(t + c_\alpha(r), r')^\lambda f_{vr}(t' + c_\alpha(r'), r')^{1-\lambda} \end{aligned}$$

for $\lambda := (t' - t)/[(t' - t) + c_\alpha(r') - c_\alpha(r)] \in (0, 1)$, since $t' + c_\alpha(r) = (1 - \lambda)(t + c_\alpha(r)) + \lambda(t' + c_\alpha(r'))$ and $t + c_\alpha(r') = \lambda(t + c_\alpha(r)) + (1 - \lambda)(t' + c_\alpha(r'))$. Combining the two inequalities, we get the first inequality. The second inequality holds because by negative affiliation, $f_{vr}(v, r)f_{vr}(v', r') \leq f_{vr}(v, r')f_{vr}(v', r)$ for $v = t + c_\alpha(r) \leq v' = t' + c_\alpha(r)$ and $r \leq r'$.

Step 4: g is non-decreasing, from $\varphi(\bar{r})$ to $\varphi(\underline{r})$. Fix $\alpha \in [0, 1]$, $t, t' \in (\underline{v} - c_\alpha(\bar{r}), \bar{v} - c_\alpha(\underline{r}))$ with $t < t'$, and for brevity, write, in this step, $a_t(r) := \bar{f}(t + c_\alpha(r), r)$, so that $f_\alpha(t) = \int_{\underline{r}}^{\bar{r}} a_t(r) dr$ and $f_\alpha(t)h_\alpha(t) = \int_{\underline{r}}^{\bar{r}} \varphi(r)a_t(r)dr$ by Step 2, and likewise for t' . Then,

$$\begin{aligned} f_\alpha(t)f_\alpha(t')h_\alpha(t') &= \int_{\underline{r}}^{\bar{r}} a_t(r)dr \int_{\underline{r}}^{\bar{r}} \varphi(r')a_{t'}(r')dr' = \iint_{[\underline{r}, \bar{r}]^2} \varphi(r')a_t(r)a_{t'}(r')drdr', \text{ and} \\ f_\alpha(t)f_\alpha(t')h_\alpha(t) &= \int_{\underline{r}}^{\bar{r}} \varphi(r)a_t(r)dr \int_{\underline{r}}^{\bar{r}} a_{t'}(r')dr' = \iint_{[\underline{r}, \bar{r}]^2} \varphi(r)a_t(r)a_{t'}(r')drdr'. \end{aligned}$$

Combining the two, we get

$$\begin{aligned} f_\alpha(t)f_\alpha(t')[h_\alpha(t') - h_\alpha(t)] &= \iint_{[\underline{r}, \bar{r}]^2} [\varphi(r') - \varphi(r)] a_t(r)a_{t'}(r')drdr' \\ &= - \iint_{[\underline{r}, \bar{r}]^2} [\varphi(r') - \varphi(r)] a_t(r')a_{t'}(r)drdr' \\ &= \frac{1}{2} \iint_{[\underline{r}, \bar{r}]^2} [\varphi(r') - \varphi(r)] [a_t(r)a_{t'}(r') - a_t(r')a_{t'}(r)] drdr' \geq 0, \end{aligned}$$

where the second line follows by the change of variables $(r, r') \mapsto (r', r)$, and the third follows from averaging the expressions in the first two lines. The inequality follows because (i) for $r < r'$ both bracketed expressions in the integrand are non-positive given that φ

is non-increasing and $a_t(r)a_{t'}(r') \leq a_t(r')a_{t'}(r)$ by Step 3; (ii) for $r > r'$ both are non-negative; and (iii) for $r = r'$ the integrand equals zero. Therefore, $h_\alpha(t) \leq h_\alpha(t')$. Also, for every α , $Q_\alpha(q)$ is increasing in q , so $q \mapsto h_\alpha(Q_\alpha(q))$ is non-decreasing for every α and thus g is non-decreasing by Step 2. By Lemma 4, $Q_0(q) \rightarrow \underline{v} - m(\bar{r})$ and $Q_1(q) \rightarrow \underline{v} - m_1(\bar{r})$ as $q \rightarrow 0$, and $Q_0(q) \rightarrow \bar{v} - m(\underline{r})$ and $Q_1(q) \rightarrow \bar{v} - m_1(\underline{r})$ as $q \rightarrow 1$, so

$$g(0^+) = \varphi(\bar{r}) = \zeta(2 - \zeta)\sigma^2 \max\{r^* - \bar{r}, 0\} < \zeta(2 - \zeta)\sigma^2 (r^* - \underline{r}) = \varphi(\underline{r}) = g(1^-),$$

the inequality by $\underline{r} < \bar{r}$ and $r^* > \underline{r}$.

Step 5: The no-waiver policy FOSD-decreases aggregate buyer surplus, strictly in expectation. By Step 4, $g(w) \leq g(q)$ at $w \leq q$, so the second formula of Step 1's display gives $R^p(q) \leq R^*(q)$ at every $q \in (0, 1)$, and, at every x ,

$$\begin{aligned} \Pr(M^p - \tilde{b}^p(M^p) \leq x) &= \Pr(R^p(q_{(N)}) \leq x) \geq \Pr(R^*(q_{(N)}) \leq x) \\ &= \Pr(M^* - \tilde{b}^*(M^*) \leq x). \end{aligned}$$

For the footnote's inequality in expectation, g is non-decreasing and not constant by Step 4, so some $0 < w_1 < q_1 < 1$ have $g(w_1) < g(q_1)$, and $g(q) - g(w) \geq g(q_1) - g(w_1)$ at $w \leq w_1$ and $q \geq q_1$; hence

$$\begin{aligned} \mathbb{E}[M^* - \tilde{b}^*(M^*)] - \mathbb{E}[M^p - \tilde{b}^p(M^p)] &= \int_0^1 \int_0^q [g(q) - g(w)] k_q(w) dw N q^{N-1} dq \\ &\geq [g(q_1) - g(w_1)] \int_{q_1}^1 \int_0^{w_1} k_q(w) dw N q^{N-1} dq \\ &= [g(q_1) - g(w_1)] N w_1^{N-1} (1 - q_1) > 0, \end{aligned}$$

the inequality because the integrand is nonnegative on $\{(q, w) : 0 \leq w \leq q \leq 1\}$ and at least $[g(q_1) - g(w_1)]k_q(w)Nq^{N-1}$ on $[q_1, 1] \times [0, w_1]$, and the equality by $\int_0^{w_1} k_q(w)dw = (w_1/q)^{N-1}$.

Step 6: A ban FOSD-increases aggregate buyer surplus, strictly in expectation if $r^* < \bar{r}$. Under a ban no offer carries a contingency, so every buyer holds $d_i = 0$, the seller ranks offers by price, and a buyer who wins at price b_i earns $v_i - b_i - \mu - r_i\sigma^2 = V(0, v_i, r_i) - b_i$: The regime is the symmetric first-price auction in $V(0, v_i, r_i) = v_i - m_0(r_i)$, m_0 affine of slope $\sigma^2 > 0$, so by Lemma 4 at m_0 and Lemma 6(iii)–(v) each buyer bids $\tilde{b}^{\text{ban}}(V(0, v_i, r_i))$, with $\tilde{b}^{\text{ban}}$ the $\tilde{b}$ of Lemma 6 at the CDF of $V(0, v_i, r_i)$, and the seller accepts the highest bid, at least $\tilde{b}^{\text{ban}}(V(0, \underline{v}, \bar{r})) = V(0, \underline{v}, \bar{r}) > 0$ by Lemma 6(iii) and Lemma 5. Write $M^{\text{ban}} := \max_i V(0, v_i, r_i)$, s^{ban} for the quantile of $V(0, v_i, r_i)$, $R^{\text{ban}}(q) := s^{\text{ban}}(q) - \tilde{b}^{\text{ban}}(s^{\text{ban}}(q))$, $g^{\text{ban}}(q) := s^*(q) - s^{\text{ban}}(q)$, and, by the proof of Lemma 1, $\varphi^{\text{ban}}(r) := m_0(r) - m(r) = \max\{m_0(r) - m_1(r), 0\} = \zeta(2 - \zeta)\sigma^2 \max\{r - r^*, 0\}$, nonnegative and non-decreasing in r . Step 1 holds at $g = \text{ban}$ by Lemma 4 at m_0 : $M^{\text{ban}} - \tilde{b}^{\text{ban}}(M^{\text{ban}})$

has the law of $R^{\text{ban}}(q_{(N)})$ and $R^{\text{ban}}(q) - R^*(q) = \int_0^q [g^{\text{ban}}(w) - g^{\text{ban}}(q)] k_q(w) dw$. Steps 2–4 hold with $c_\alpha^{\text{ban}}(r) := m(r) + \alpha \varphi^{\text{ban}}(r) = (1 - \alpha)m(r) + \alpha m_0(r)$ in place of c_α , continuous and increasing in r for $\alpha \in (-\varepsilon', 1 + \varepsilon')$ given $\varepsilon' \in (0, (1 - \zeta)^2 / [\zeta(2 - \zeta)])$, its slope being σ^2 for $r < r^*$ and $[(1 - \zeta)^2 + \alpha \zeta(2 - \zeta)]\sigma^2 > 0$ for $r > r^*$, and φ^{ban} in place of φ , with $Q_0^{\text{ban}} = s^*$, $Q_1^{\text{ban}} = s^{\text{ban}}$ and $h_\alpha^{\text{ban}}(t) := \mathbb{E}[\varphi^{\text{ban}}(r_i) | v_i - c_\alpha^{\text{ban}}(r_i) = t]$, except that φ^{ban} is non-decreasing: In Step 4's second display the first bracket is nonnegative at $r < r'$ and nonpositive at $r > r'$, so the integrand is nonpositive, $h_\alpha^{\text{ban}}(t) \geq h_\alpha^{\text{ban}}(t')$ at $t < t'$, and $g^{\text{ban}}(q) = \int_0^1 h_\alpha^{\text{ban}}(Q_\alpha^{\text{ban}}(q)) d\alpha$ is non-increasing on $(0, 1)$, with $g^{\text{ban}}(0^+) = \varphi^{\text{ban}}(\bar{r}) = \zeta(2 - \zeta)\sigma^2 \max\{\bar{r} - r^*, 0\}$ and $g^{\text{ban}}(1^-) = \varphi^{\text{ban}}(\underline{r}) = 0$. Hence $R^{\text{ban}}(q) \geq R^*(q)$ at every $q \in (0, 1)$ and $\Pr(M^{\text{ban}} - \tilde{b}^{\text{ban}}(M^{\text{ban}}) \leq x) \leq \Pr(M^* - \tilde{b}^*(M^*) \leq x)$ at every x . If $r^* < \bar{r}$, then $g^{\text{ban}}(0^+) > g^{\text{ban}}(1^-)$, some $0 < w_1 < q_1 < 1$ have $g^{\text{ban}}(w_1) > g^{\text{ban}}(q_1)$, and, as in Step 5,

$$\begin{aligned} \mathbb{E}[M^{\text{ban}} - \tilde{b}^{\text{ban}}(M^{\text{ban}})] - \mathbb{E}[M^* - \tilde{b}^*(M^*)] &= \int_0^1 \int_0^q [g^{\text{ban}}(w) - g^{\text{ban}}(q)] k_q(w) dw N q^{N-1} dq \\ &\geq [g^{\text{ban}}(w_1) - g^{\text{ban}}(q_1)] N w_1^{N-1} (1 - q_1) > 0. \end{aligned}$$

Step 7: A ban FOSD-decreases total and seller surplus. $V^*(v_j, r_j) \geq V(0, v_j, r_j)$ for every j and every (v_j, r_j) , since $V^*(v_j, r_j) = \max_{d \in \{0, 1\}} V(d, v_j, r_j)$, so Part (i) of the proof of Proposition 3, with $V(0, v_j, r_j)$ and M^{ban} in place of $V(1, v_j, r_j)$ and M^{p} , gives $M^{\text{ban}} \leq M^*$ for every realization $(v_i, r_i)_{i \in \mathcal{N}}$ of the buyers' types. Steps 1–3 of Part (ii) of that proof use only that m_1 is continuous and increasing with $m_1 \geq m$, and Lemmas 4 and 6; m_0 is continuous and increasing with $m_0 - m = \varphi^{\text{ban}} \geq 0$, and both lemmas apply at m_0 as in Step 6. So those steps hold with m_0 , the CDF of $V(0, v_i, r_i)$, s^{ban} and $\tilde{b}^{\text{ban}}$ in place of m_1 , F_V^{p} , s^{p} and $\tilde{b}^{\text{p}}$, and give $\Pr(\tilde{b}^{\text{ban}}(M^{\text{ban}}) \leq x) \geq \Pr(\tilde{b}^*(M^*) \leq x)$ for every x . **Q.E.D.**

Proof of Proposition 5. Write, in this proof, $\bar{\varphi} := \varphi(\underline{r})$ for the largest distortion, $\underline{s} := V^*(\underline{v}, \bar{r})$, $\underline{s}^{\text{p}} := V(1, \underline{v}, \bar{r})$, $\check{s} := V^*(\underline{v}, r^\varphi)$, and $A^{\text{g}}(x) := \int_{\underline{s}^{\text{p}}}^x F_V^{\text{g}}(t)^{N-1} dt$, $\text{g} \in \{*, \text{p}\}$.

Step 1: As in the proof of Proposition 1, $V^*(v_i, r_i) = v_i - m(r_i)$, $V(1, v_i, r_i) = v_i - m_1(r_i)$, and $V(1, v_i, r_i) = V^*(v_i, r_i) - \varphi(r_i)$. Since $\underline{r} < r^*$, $\bar{\varphi} = \zeta(2 - \zeta)\sigma^2(r^* - \underline{r}) > 0$ and $\varphi(\bar{r}) = \zeta(2 - \zeta)\sigma^2(r^* - r^\varphi) \geq 0$, so $\underline{s}^{\text{p}} = \underline{v} - m_1(\bar{r}) = \underline{s} - \varphi(\bar{r})$. Also, $\check{s} = \underline{v} - m(r^\varphi)$, equal to $\underline{v} - m_1(r^*)$ if $r^* \leq \bar{r}$, since $\varphi(r^*) = 0$, and $\underline{s} \leq \check{s} < V^*(\bar{v}, \underline{r})$, the first with equality if and only if $r^* \geq \bar{r}$, given that m is increasing, $\underline{v} < \bar{v}$, and $\underline{r} < r^\varphi \leq \bar{r}$. By Lemma 4, each A^{g} is continuous and non-decreasing, $A^*(x) = 0$ for $x \leq \underline{s}$ and $A^{\text{p}}(x) = 0$ for $x \leq \underline{s}^{\text{p}}$, and each is increasing above that point. Then, $u^*(v_i, r_i) = A^*(V^*(v_i, r_i))$ and $u^{\text{p}}(v_i, r_i) = A^{\text{p}}(V^*(v_i, r_i) - \varphi(r_i))$, so $\Delta u^{\text{p}}(v_i, r_i) = A^{\text{p}}(V^*(v_i, r_i) - \varphi(r_i)) - A^*(V^*(v_i, r_i))$.

$F_V^{\text{p}}(t) \geq F_V^*(t)$ for every t with strict inequality for $t \in (\check{s}, V^*(\bar{v}, \underline{r}))$. More generally, $F_V^{\text{p}}(t - \varphi(\bar{r})) \geq F_V^*(t)$ for every t with strict inequality for $t \in (\check{s}, V^*(\bar{v}, \underline{r}))$: If $r^* \leq \bar{r}$, $\varphi(\bar{r}) = 0$ and this is the previous sentence; if $r^* > \bar{r}$, then $\check{s} = \underline{s}$, $F_V^{\text{p}}(t - \varphi(\bar{r})) - F_V^*(t) = \Pr(t < v_j - m(r_j) \leq t + \varphi(r_j) - \varphi(\bar{r}))$ with $\varphi(r_j) - \varphi(\bar{r}) = \zeta(2 - \zeta)\sigma^2(\bar{r} - r_j)$, and at

$t \in (\underline{s}, V^*(\bar{v}, \underline{r}))$ some type (v_0, r_0) in the interior of the type rectangle has $v_0 - m(r_0) = t$, so the types near $(v_0 + \varepsilon, r_0)$, for small $\varepsilon > 0$, form an open set inside the event, which therefore has positive probability, f_{vr} being positive. Also, since $m_1 = m + \varphi$ and $\varphi(r_j) \leq \bar{\varphi}$, for every $r_j \in [\underline{r}, \bar{r}]$, it holds that $F_V^p(t - \bar{\varphi}) = \Pr(v_j - m(r_j) \leq t - [\bar{\varphi} - \varphi(r_j)]) \leq F_V^*(t)$ for every t . We also have that for any effective valuation s ,

$$A^p(s - \bar{\varphi}) = \int_{\underline{s}^p}^{s - \bar{\varphi}} F_V^p(w)^{N-1} dw = \int_{\underline{s}^p + \bar{\varphi}}^s F_V^p(t - \bar{\varphi})^{N-1} dt = \int_{\underline{s}}^s F_V^p(t - \bar{\varphi})^{N-1} dt.$$

Also, $A^p(s - \varphi(\bar{r})) - A^*(s) = \int_{\underline{s}}^s [F_V^p(t - \varphi(\bar{r}))^{N-1} - F_V^*(t)^{N-1}] dt = 0$ for $s \in [\underline{s}, \check{s}]$, $A^p(s - \varphi(\bar{r})) - A^*(s) > 0$ for $s \in (\check{s}, V^*(\bar{v}, \underline{r})]$, and $A^*(s) - A^p(s - \bar{\varphi}) = \int_{\underline{s}}^s [F_V^*(t)^{N-1} - F_V^p(t - \bar{\varphi})^{N-1}] dt > 0$ for $s \in (\underline{s}, V^*(\bar{v}, \underline{r})]$.

Step 2: Write $(A^p)^{-1}$ for the (increasing and continuous) inverse of A^p restricted to $[\underline{s}^p, \infty)$, continuous and increasing on $[0, \infty)$. For every $s \in [\underline{s}, V^*(\bar{v}, \underline{r})]$, define $\chi(s) := s - (A^p)^{-1}(A^*(s))$ and $r^\dagger(s) := r^* - \chi(s)/[\zeta(2 - \zeta)\sigma^2]$, so that $A^p(s - \chi(s)) = A^*(s)$ for $s - \chi(s) \geq \underline{s}^p$. $\chi(s)$ is the distortion that would leave a buyer (v_i, r_i) with effective valuation $V^*(v_i, r_i) = s$ exactly as well off under the policy as without it. $r^\dagger(s)$ is that threshold type's r_i . That is, $V^*(v_i, r_i) = s$ and $A^p(s - \chi(s)) = A^*(s)$.

We have that $\varphi(\bar{r}) \leq \chi(s) < \bar{\varphi}$, and given Step 1, (i) $\chi(s) = \varphi(\bar{r})$ if and only if $s \leq \check{s}$ and (ii) $s - \chi(s) = \underline{s}^p$ if and only if $s = \underline{s}$. Since $\bar{\varphi} = \zeta(2 - \zeta)\sigma^2(r^* - \underline{r})$ and $\varphi(\bar{r}) = \zeta(2 - \zeta)\sigma^2(r^* - r^\varphi)$, $r^\dagger(s) \in (\underline{r}, r^\varphi]$. Also, $r^\dagger(\cdot)$ is continuous since A^* and $(A^p)^{-1}$ are so. $\varphi(r^\dagger(s)) = \chi(s)$, since $r^\dagger(s) \leq r^*$. Given that $\varphi(r)$ is decreasing on $[\underline{r}, r^\varphi]$ and $\varphi(r) = \varphi(\bar{r})$ for $r \in [r^\varphi, \bar{r}]$, for every $r_i \in [\underline{r}, \bar{r}]$ and every s with $\chi(s) > \varphi(\bar{r})$, the following equivalences hold: (i) $\varphi(r_i) > \chi(s) \iff r_i < r^\dagger(s)$, (ii) $\varphi(r_i) = \chi(s) \iff r_i = r^\dagger(s)$, and (iii) $\varphi(r_i) < \chi(s) \iff r_i > r^\dagger(s)$.

Step 3: For $V^*(v_i, r_i) > \check{s}$, Step 2 gives $\chi(V^*(v_i, r_i)) > \varphi(\bar{r})$ and $V^*(v_i, r_i) - \chi(V^*(v_i, r_i)) > \underline{s}^p$, so given that from Steps 1 and 2, $\Delta u^p(v_i, r_i) = A^p(V^*(v_i, r_i) - \varphi(r_i)) - A^*(V^*(v_i, r_i)) = A^p(V^*(v_i, r_i) - \varphi(r_i)) - A^p(V^*(v_i, r_i) - \chi(V^*(v_i, r_i)))$, the following equivalences hold: (i) $\Delta u^p(v_i, r_i) < 0 \iff \varphi(r_i) > \chi(s) \iff r_i < r^\dagger(V^*(v_i, r_i))$, (ii) $\Delta u^p(v_i, r_i) = 0 \iff \varphi(r_i) = \chi(V^*(v_i, r_i)) \iff r_i = r^\dagger(V^*(v_i, r_i))$, and (iii) $\Delta u^p(v_i, r_i) > 0 \iff \varphi(r_i) < \chi(V^*(v_i, r_i)) \iff r_i > r^\dagger(V^*(v_i, r_i))$.

For every (v_i, r_i) with $V^*(v_i, r_i) \leq \check{s} \equiv V^*(\underline{v}, r^\varphi)$, it holds that $r_i \geq r^\varphi$, so that $\varphi(r_i) = \varphi(\bar{r})$, φ being constant on $[r^\varphi, \bar{r}]$, $\chi(V^*(v_i, r_i)) = \varphi(\bar{r})$ by Step 2, and $r^\dagger(s) = r^\varphi$. Therefore, $\Delta u^p(v_i, r_i) = A^p(s - \varphi(\bar{r})) - A^*(s) = 0$. Combining, we conclude that: (i) $\Delta u^p(v_i, r_i) < 0 \iff r_i < r^\dagger(s)$, (ii) $\Delta u^p(v_i, r_i) > 0 \iff [r_i > r^\dagger(s) \wedge s > V^*(\underline{v}, r^\varphi)]$, and (iii) $\Delta u^p(v_i, r_i) = 0 \iff [r_i = r^\dagger(s) \vee s \leq V^*(\underline{v}, r^\varphi)]$. If $r^\varphi = \bar{r}$, the set $\{V^*(v_i, r_i) \leq \check{s}\}$ is the single type $(\underline{v}, \bar{r})$, and the contour $\{r_i = r^\dagger(V^*(v_i, r_i))\}$ is the image of the graph of the continuous $r^\dagger(\cdot)$ under the Lipschitz map $(s, r) \mapsto (s + m(r), r)$ and so of Lebesgue measure zero, which gives the footnote to the statement. **Q.E.D.**

Proof of Proposition 6. Step 1 shows that a buyer chooses the contingency term and the share to maximize the surplus of the sale, Step 2 identifies the auction that leaves, and Step 3 computes realized total surplus.

Step 1: The contingency term and the share maximize the surplus of the sale. The seller's payoff from accepting offer (b_i, d_i, λ_i) is, in this proof, $\tilde{b}_i := b_i - d_i[\kappa_S + \lambda_i\mu + \lambda_i^2 r_S \sigma^2]$, so she accepts the largest $\tilde{b}_i$, and buyer i 's acceptance event turns on $\tilde{b}_i$ and the rivals' offers alone. If his offer is accepted, buyer i pays b_i , $d_i\kappa_B$ and his share $(1 - \lambda_i d_i)c$ of the cost c , so that, by $\kappa_B + \kappa_S = \kappa$ and $d_i^2 = d_i$, he earns

$$\begin{aligned} v_i - b_i - d_i\kappa_B - (1 - \lambda_i d_i)\mu - (1 - \lambda_i d_i)^2 r_i \sigma^2 \\ = v_i - d_i\kappa - \mu - \left[(1 - \lambda_i d_i)^2 r_i + \lambda_i^2 d_i r_S \right] \sigma^2 - \tilde{b}_i. \end{aligned}$$

At $d_i = 0$ the right-hand side is $V(0, v_i, r_i) - \tilde{b}_i$. At $d_i = 1$, $(1 - \lambda_i)^2 r_i + \lambda_i^2 r_S$ is a convex quadratic in λ_i with derivative $2[\lambda_i r_S - (1 - \lambda_i)r_i]$ and second derivative $2(r_i + r_S)$, positive when $r_i + r_S > 0$, minimized at $\lambda_i = \zeta^{\text{ss}}(r_i)$, where it equals $(r_S^2 r_i + r_i^2 r_S)/(r_i + r_S)^2 = r_i r_S/(r_i + r_S)$; at $r_i = r_S = 0$ it is 0 at every λ_i , as is $r_i r_S/(r_i + r_S)$ under the footnote's convention of section 3.1.4. So at $d_i = 1$ the right-hand side is at most $v_i - \kappa - \mu - r_i r_S \sigma^2/(r_i + r_S) - \tilde{b}_i$, with equality if and only if $\lambda_i = \zeta^{\text{ss}}(r_i)$ when $r_i + r_S > 0$, and at a fixed $\tilde{b}_i$ the maximum over $d_i \in \{0, 1\}$ and $\lambda_i \in [0, 1]$ is $V^{\text{ss}}(v_i, r_i) - \tilde{b}_i$. At $r_i + r_S > 0$, $V^{\text{ss}}(v_i, r_i) > V(0, v_i, r_i)$ if and only if $r_i \sigma^2 - r_i r_S \sigma^2/(r_i + r_S) > \kappa$, that is, $r_i^2 \sigma^2 - \kappa r_i - \kappa r_S > 0$; this convex quadratic in r_i has the roots $[\kappa \pm (\kappa^2 + 4\kappa r_S \sigma^2)^{1/2}]/(2\sigma^2)$, the smaller nonpositive and the larger $r^{\text{ss}} = [\kappa + (\kappa^2 + 4\kappa r_S \sigma^2)^{1/2}]/(2\sigma^2)$, so it is positive exactly at $r_i > r^{\text{ss}}$ on $[0, \infty)$; and at $r_i = r_S = 0$ both $V^{\text{ss}}(v_i, r_i) > V(0, v_i, r_i)$ and $r_i > r^{\text{ss}}$ are false, by the footnote's convention and $r^{\text{ss}} \geq 0$. The maximum is therefore attained at $d_i = d^{\text{ss}}(r_i)$ and, if $d_i = 1$, $\lambda_i = \zeta^{\text{ss}}(r_i)$, uniquely if $r_i \neq r^{\text{ss}}$, since $r_i > r^{\text{ss}} \geq 0$ implies $r_i + r_S > 0$. With $P > 0$ his acceptance probability, replacing any other contingency term, or any other share in a contingent offer, by these at the same $\tilde{b}_i$ leaves P unchanged and raises his expected payoff by P times a positive amount when $r_i \neq r^{\text{ss}}$, so in an equilibrium in which he wins with positive probability he offers them.

Step 2: The auction. By Step 1, buyer i 's payoff from winning at the effective bid $\tilde{b}_i$ is $V^{\text{ss}}(v_i, r_i) - \tilde{b}_i$, with $V^{\text{ss}}(v_i, r_i) = v_i - m^{\text{ss}}(r_i)$, where $m^{\text{ss}}(r) := \mu + \min\{r\sigma^2, \kappa + r r_S \sigma^2/(r + r_S)\}$ is continuous and non-decreasing in r , the second argument having derivative $r_S^2 \sigma^2/(r + r_S)^2 \geq 0$. By Lemma 4, F_V^{ss} is continuous and increasing on $[V^{\text{ss}}(\underline{v}, \bar{r}), V^{\text{ss}}(\bar{v}, \underline{r})]$, so by Lemma 6(iii)–(v) the unique symmetric monotone equilibrium effective bid is $\tilde{b}^{\text{ss}}(V^{\text{ss}}(v_i, r_i))$, which buyer i reaches with the contingency term $d^{\text{ss}}(r_i)$ and the share $\zeta^{\text{ss}}(r_i)$ at the price $b^{\text{ss}}(v_i, r_i) = \tilde{b}^{\text{ss}}(V^{\text{ss}}(v_i, r_i)) + d^{\text{ss}}(r_i)\omega^{\text{ss}}(r_i)$, with $\omega^{\text{ss}}(r_i) = \kappa_S + \zeta^{\text{ss}}(r_i)\mu + \zeta^{\text{ss}}(r_i)^2 r_S \sigma^2$. The highest effective bid is at least $\tilde{b}^{\text{ss}}(V^{\text{ss}}(\underline{v}, \bar{r})) = V^{\text{ss}}(\underline{v}, \bar{r}) \geq V(0, \underline{v}, \bar{r}) > 0$, by Lemma 6(iii) and Lemma 5, so the seller accepts it, and the buyer with the highest $V^{\text{ss}}(v_i, r_i)$ wins.

Step 3: Realized total surplus. At the winner, the buyer earns $V^{\text{ss}}(v_i, r_i) - \tilde{b}^{\text{ss}}(V^{\text{ss}}(v_i, r_i))$

and the seller $\tilde{b}^{\text{ss}}(V^{\text{ss}}(v_i, r_i))$, which sum to $V^{\text{ss}}(v_i, r_i) = M^{\text{ss}}$.

Q.E.D.

Proof of Proposition 7. *Part (i).* By Step 1 of the proof of Proposition 6 at $\lambda_i = \zeta$, $V(1, v_i, r_i) = v_i - \kappa - \mu - [(1 - \zeta)^2 r_i + \zeta^2 r_S] \sigma^2 \leq v_i - \kappa - \mu - r_i r_S \sigma^2 / (r_i + r_S)$, with equality if and only if $\zeta = \zeta^{\text{ss}}(r_i)$ when $r_i + r_S > 0$, so $V^{\text{ss}}(v_i, r_i) \geq \max\{V(0, v_i, r_i), V(1, v_i, r_i)\} = V^*(v_i, r_i)$ for every (v_i, r_i) . If $r_i \leq r^{\text{ss}}$, then $V(1, v_i, r_i) \leq V(0, v_i, r_i) = V^{\text{ss}}(v_i, r_i)$, so $V^{\text{ss}}(v_i, r_i) = V^*(v_i, r_i)$. If $r_i > r^{\text{ss}}$, then $r_i + r_S > 0$ and $V^{\text{ss}}(v_i, r_i) > V(0, v_i, r_i)$, so $V^{\text{ss}}(v_i, r_i) = V^*(v_i, r_i)$ if and only if $\zeta = \zeta^{\text{ss}}(r_i)$. Taking the maximum over i , $M^{\text{ss}} \geq M^*$. $M^{\text{ss}} = M^*$ if and only if for some i with $V^{\text{ss}}(v_i, r_i) = M^{\text{ss}}$, $V^{\text{ss}}(v_i, r_i) = V^*(v_i, r_i)$. Finally, for $r_i > r^*$, it holds that $V^{\text{ss}}(v_i, r_i) \geq V(1, v_i, r_i) > V(0, v_i, r_i)$, so $r_i > r^{\text{ss}}$. Thus, $r^{\text{ss}} \leq r^*$. If $r^* > 0$, $V(1, v_i, r^*) = V(0, v_i, r^*)$. Given that r^{ss} is the larger root of $r^2 \sigma^2 - \kappa r - \kappa r_S$, whose roots have product $-\kappa r_S / \sigma^2 \leq 0$, $r^{\text{ss}} = r^*$ if and only if $v_i - \kappa - \mu - r^* r_S \sigma^2 / (r^* + r_S) = V(0, v_i, r^*) = V(1, v_i, r^*)$, that is, if and only if $\zeta = \zeta^{\text{ss}}(r^*)$.

Part (ii): Seller surplus. Step 1 writes the market's distortion of effective valuations as a function of r_i and signs its slope, Step 2 proves part (ii), and Step 3 proves the footnote's strict inequality for seller surplus.

Step 1: The distortion and its slope. Recall m_0 , m_1 and $m = \min\{m_0, m_1\}$ from the proof of Proposition 1 and m^{ss} from Step 2 of the proof of Proposition 6, write $m_1^{\text{ss}}(r) := \kappa + \mu + r r_S \sigma^2 / (r + r_S)$, so that $m^{\text{ss}} = \min\{m_0, m_1^{\text{ss}}\}$, and write $\varphi^{\text{ss}}(r) := m^{\text{ss}}(r) - m(r)$, so that $V^{\text{ss}}(v_i, r_i) = V^*(v_i, r_i) - \varphi^{\text{ss}}(r_i)$. By the proof of Lemma 1, $m = m_0$ on $r \leq r^*$ and $m = m_1$ on $r \geq r^*$; by Step 1 of the proof of Proposition 6, $m^{\text{ss}} = m_0$ on $r \leq r^{\text{ss}}$ and $m^{\text{ss}} = m_1^{\text{ss}}$ on $r \geq r^{\text{ss}}$; and $r^{\text{ss}} \leq r^*$ by Part (i). Hence, by $(1 - \zeta)^2 r + \zeta^2 r_S - r r_S / (r + r_S) = (r + r_S)(\zeta - \zeta^{\text{ss}}(r))^2$,

$$-\varphi^{\text{ss}}(r) = \begin{cases} 0 & \text{if } r \leq r^{\text{ss}}, \\ r \sigma^2 - \kappa - r r_S \sigma^2 / (r + r_S) & \text{if } r^{\text{ss}} < r < r^*, \\ (r + r_S)(\zeta - \zeta^{\text{ss}}(r))^2 \sigma^2 & \text{if } r \geq r^*, \end{cases}$$

continuous, the second row vanishing at r^{ss} and the last two agreeing at r^* , where $m_0 = m_1$. The second row has derivative $\sigma^2[1 - r_S^2 / (r + r_S)^2] > 0$ at $r > 0$. With $\zeta^{\text{ss}'}(r) = r_S / (r + r_S)^2$ and $1 - \zeta^{\text{ss}}(r) = r_S / (r + r_S)$, the last row has derivative

$$[(\zeta - \zeta^{\text{ss}}(r))^2 - 2(\zeta - \zeta^{\text{ss}}(r))(1 - \zeta^{\text{ss}}(r))] \sigma^2 = (\zeta - \zeta^{\text{ss}}(r))(\zeta + \zeta^{\text{ss}}(r) - 2) \sigma^2,$$

whose sign is that of $\zeta^{\text{ss}}(r) - \zeta$, since $\zeta < 1$ and $\zeta^{\text{ss}}(r) \leq 1$. ζ^{ss} is increasing if $r_S > 0$ and equal to 1 at every $r > 0$ if $r_S = 0$. Whatever ζ , if $r^{\text{ss}} < \bar{r}$, then $\varphi^{\text{ss}} < 0$ on $(\max\{r^{\text{ss}}, \underline{r}\}, \bar{r}]$ except at most at one r , the one with $\zeta^{\text{ss}}(r) = \zeta$, the second row being positive and the last row vanishing only there.

Step 2: Seller surplus. $\varphi^{\text{ss}} \leq 0$, so at every t , $F_V^*(t) - F_V^{\text{ss}}(t) = \Pr(V^*(v_i, r_i) \leq t <$

$V^{\text{ss}}(v_i, r_i) \geq 0$, and $s^{\text{ss}}(q) \geq s^*(q)$ at every $q \in (0, 1]$, with s^{ss} the quantile of $V^{\text{ss}}(v_i, r_i)$. By Lemma 6(i), at every $q \in (0, 1]$,

$$\tilde{b}^{\text{ss}}(s^{\text{ss}}(q)) - \tilde{b}^*(s^*(q)) = \int_0^q [s^{\text{ss}}(w) - s^*(w)] \frac{(N-1)w^{N-2}}{q^{N-1}} dw \geq 0.$$

Step 3 of the proof of Proposition 3 uses only that F_V^g and $\tilde{b}^g$ are continuous and increasing, which holds at $g = \text{ss}$ by Step 2 of the proof of Proposition 6; with $q^g(x) := F_V^g((\tilde{b}^g)^{-1}(x))$ it gives $\Pr(\tilde{b}^g(M^g) \leq x) = q^g(x)^N$ and $\tilde{b}^g(s^g(q)) \leq x \iff q \leq q^g(x)$ at $g \in \{*, \text{ss}\}$. At $q^{\text{ss}}(x) = 0$ the claim holds outright; at $q^{\text{ss}}(x) > 0$,

$$\begin{aligned} \tilde{b}^*(s^*(q^{\text{ss}}(x))) \leq \tilde{b}^{\text{ss}}(s^{\text{ss}}(q^{\text{ss}}(x))) \leq x &\implies q^{\text{ss}}(x) \leq q^*(x) \\ &\implies \Pr(\tilde{b}^{\text{ss}}(M^{\text{ss}}) \leq x) \leq \Pr(\tilde{b}^*(M^*) \leq x). \end{aligned}$$

Step 3: The footnote's strict inequality for seller surplus. Write $\hat{r} := \max\{r^{\text{ss}}, \underline{r}\}$, less than $\bar{r}$, $r^{\text{ss}} < \bar{r}$ being maintained in section 3.1.4, $\underline{s} := V^*(\underline{v}, \bar{r})$ and $J := (\underline{s}, V^{\text{ss}}(\bar{v}, \hat{r}))$, non-empty because $V^{\text{ss}}(\bar{v}, \hat{r}) \geq V^*(\bar{v}, \hat{r}) > V^*(\underline{v}, \bar{r})$, by $\underline{v} < \bar{v}$, $\hat{r} < \bar{r}$ and m being increasing. We first show that $F_V^*(t) - F_V^{\text{ss}}(t) = \Pr(V^*(v_j, r_j) \leq t < V^{\text{ss}}(v_j, r_j))$ is positive if and only if $t \in J$, taking the three positions of t in turn.

1. If $t \leq \underline{s}$, then $V^*(v_j, r_j) \leq t$ requires $V^*(v_j, r_j) = \underline{s}$, that is, $(v_j, r_j) = (\underline{v}, \bar{r})$, m being increasing, which happens with probability 0.
2. If $t \geq V^{\text{ss}}(\bar{v}, \hat{r})$, then $V^*(v_j, r_j) \leq t < V^{\text{ss}}(v_j, r_j)$ requires $\varphi^{\text{ss}}(r_j) < 0$, so $r_j > r^{\text{ss}}$ by Step 1 and $r_j \geq \hat{r}$; but then $V^{\text{ss}}(v_j, r_j) = v_j - m^{\text{ss}}(r_j) \leq \bar{v} - m^{\text{ss}}(\hat{r}) \leq t$, m^{ss} being non-decreasing. The probability is 0.
3. Let $t \in J$. At $r \in [\underline{r}, \bar{r}]$ the valuations v with $V^*(v, r) \leq t < V^{\text{ss}}(v, r)$ form $(t + m^{\text{ss}}(r), t + m(r)] \cap [\underline{v}, \bar{v}]$, a non-degenerate interval if $t + m^{\text{ss}}(r) < \bar{v}$, $t + m(r) > \underline{v}$ and $\varphi^{\text{ss}}(r) < 0$. Write $r_c := \sup\{r \in [\hat{r}, \bar{r}] : t + m^{\text{ss}}(r) < \bar{v}\}$; m^{ss} being continuous and non-decreasing, $r_c > \hat{r}$, by $t + m^{\text{ss}}(\hat{r}) < \bar{v}$, which is $t \in J$, and $t + m^{\text{ss}}(r) < \bar{v}$ at every $r \in [\hat{r}, r_c)$. Either $r_c = \bar{r}$, and then $t + m(r_c) > \underline{v}$ by $t > \underline{s} = \underline{v} - m(\bar{r})$; or $t + m^{\text{ss}}(r_c) = \bar{v}$, and then $t + m(r_c) \geq t + m^{\text{ss}}(r_c) = \bar{v} > \underline{v}$, by $\varphi^{\text{ss}} \leq 0$. Either way $t + m(r_c) > \underline{v}$, so, m being continuous, some $r_b \in [\hat{r}, r_c)$ has $t + m(r) > \underline{v}$ and $t + m^{\text{ss}}(r) < \bar{v}$ at every $r \in (r_b, r_c)$, where also $\varphi^{\text{ss}}(r) < 0$ except at most at one r , by Step 1 and $\hat{r} \geq r^{\text{ss}}$. Hence

$$\Pr(V^*(v_j, r_j) \leq t < V^{\text{ss}}(v_j, r_j)) \geq \int_{r_b}^{r_c} \int_{\max\{t+m^{\text{ss}}(r), \underline{v}\}}^{\min\{t+m(r), \bar{v}\}} f_{vr}(v, r) dv dr > 0,$$

the region of integration lying inside the event and being non-degenerate at almost every $r \in (r_b, r_c)$, and f_{vr} being positive.

Write $w_1 := F_V^*(V^{\text{ss}}(\bar{v}, \hat{r}))$, positive because $V^{\text{ss}}(\bar{v}, \hat{r}) > \underline{s}$ and F_V^* is increasing on its support. At $w \in (0, w_1)$, $s^*(w) > \underline{s}$, by $w > 0$, and $s^*(w) < V^{\text{ss}}(\bar{v}, \hat{r})$, by $w < w_1$, so $s^*(w) \in J$, $F_V^{\text{ss}}(s^*(w)) < F_V^*(s^*(w)) = w$, and $s^{\text{ss}}(w) > s^*(w)$, F_V^{ss} being continuous and increasing on its support. The integrand of Step 2's display is therefore nonnegative on $(0, q)$ and positive on $(0, \min\{q, w_1\})$, so $\tilde{b}^{\text{ss}}(s^{\text{ss}}(q)) > \tilde{b}^*(s^*(q))$ at every $q \in (0, 1]$. With $q^g(x)$ as in Step 2, take the three positions of x in turn. If $x \leq \underline{s}$, then $\Pr(\tilde{b}^*(M^*) \leq x) = 0$, by $\tilde{b}^*(\underline{s}) = \underline{s}$ (Lemma 6(iii)), $\tilde{b}^*$ being increasing and $M^* > \underline{s}$ almost surely, so both probabilities are 0 by Step 2. If $x \geq \tilde{b}^{\text{ss}}(V^{\text{ss}}(\bar{v}, \underline{r}))$, the largest effective bid under the policy, both are 1 by Step 2. If $\underline{s} < x < \tilde{b}^{\text{ss}}(V^{\text{ss}}(\bar{v}, \underline{r}))$, then $q^{\text{ss}}(x) < 1$, $\tilde{b}^{\text{ss}}$ being continuous and increasing; if $q^*(x) = 1$, then $q^{\text{ss}}(x) < q^*(x)$ already; and if $q^*(x) < 1$, then $q^*(x) > 0$, by $x > \underline{s} = \tilde{b}^*(\underline{s})$, and $\tilde{b}^*(s^*(q^*(x))) = x$, $\tilde{b}^*$ and F_V^* being continuous and increasing, so

$$\tilde{b}^{\text{ss}}(s^{\text{ss}}(q^*(x))) > \tilde{b}^*(s^*(q^*(x))) = x \implies q^{\text{ss}}(x) < q^*(x),$$

by (1) at $g = \text{ss}$. Either way, $\Pr(\tilde{b}^{\text{ss}}(M^{\text{ss}}) \leq x) = q^{\text{ss}}(x)^N < q^*(x)^N = \Pr(\tilde{b}^*(M^*) \leq x)$, which is the footnote's strict inequality on $(V^*(\underline{v}, \bar{r}), \tilde{b}^{\text{ss}}(V^{\text{ss}}(\bar{v}, \underline{r})))$. **Q.E.D.**

Proof of Proposition 8. Step 1 shows that, on $[\underline{r}, \bar{r}]$, φ^{ss} is non-increasing under assumption 3(a), increasing under assumption 3(b), and not monotone if neither holds; Step 2 carries the proof of Proposition 4 over to the market.

Step 1: The distortion's monotonicity under assumption 3. Recall from Step 1 of the proof of Proposition 7 the distortion φ^{ss} and the display there, whose three rows give $-\varphi^{\text{ss}}$ on $r \leq r^{\text{ss}}$, on $r^{\text{ss}} < r < r^*$ and on $r \geq r^*$: φ^{ss} is continuous, equal to 0 at $r \leq r^{\text{ss}}$, with derivative $-\sigma^2[1 - r_S^2/(r + r_S)^2] < 0$ on (r^{ss}, r^*) , where $r > 0$, and a derivative of the sign of $\zeta - \zeta^{\text{ss}}(r)$ on (r^*, ∞) ; and ζ^{ss} is increasing if $r_S > 0$ and equal to 1 at every $r > 0$ if $r_S = 0$. Therefore:

- (a) under assumption 3(a), $\zeta \leq \zeta^{\text{ss}}(r)$ at every $r \in (\max\{r^*, \underline{r}\}, \bar{r})$, this interval being empty if $r^* \geq \bar{r}$ and otherwise $\zeta \leq \zeta^{\text{ss}}(\max\{r^*, \underline{r}\}) \leq \zeta^{\text{ss}}(r)$ there, ζ^{ss} being non-decreasing, with equality at most at one r there, so, the first two rows covering $[\underline{r}, \max\{r^*, \underline{r}\}]$ and φ^{ss} being continuous, φ^{ss} is non-increasing on $[\underline{r}, \bar{r}]$ and decreasing on $[\max\{r^{\text{ss}}, \underline{r}\}, \bar{r}]$;
- (b) under assumption 3(b), $r^* \leq \underline{r}$ and $\zeta \geq \zeta^{\text{ss}}(\bar{r})$, so $r_S > 0$, since $\zeta < 1$, and $\zeta^{\text{ss}}(r) < \zeta^{\text{ss}}(\bar{r}) \leq \zeta$ at every $r \in [\underline{r}, \bar{r})$, so φ^{ss} is increasing on $[\underline{r}, \bar{r}]$.

If neither part of assumption 3 holds, φ^{ss} is not monotone on $[\underline{r}, \bar{r}]$. Indeed, assumption 3(a) failing gives $r^* < \bar{r}$ and $\zeta > \zeta^{\text{ss}}(\max\{r^*, \underline{r}\})$, so, ζ^{ss} being continuous, $\zeta > \zeta^{\text{ss}}(r_1)$ at some $r_1 \in (\max\{r^*, \underline{r}\}, \bar{r})$, where φ^{ss} is increasing. Since assumption 3(a) holds whenever $r_S = 0$, also $r_S > 0$; hence $r^* > 0$ and $\zeta > \zeta^{\text{ss}}(\max\{r^*, \underline{r}\}) \geq \zeta^{\text{ss}}(r^*)$, and $r^{\text{ss}} < r^*$ by Part (i) of the proof of Proposition 7. Then assumption 3(b) failing gives either $r^* > \underline{r}$, and φ^{ss}

is decreasing on the non-empty $(\max\{r^{\text{ss}}, \underline{r}\}, r^*)$, or $\zeta < \zeta^{\text{ss}}(\bar{r})$, and φ^{ss} is decreasing near $\bar{r}$.

Step 2: Aggregate buyer surplus. Recall φ^{ss} from Step 1 of this proof, with its monotonicity under assumptions 3(a) and 3(b). Steps 1–4 of the proof of Proposition 4 hold with the market in place of the no-waiver policy: $s^{\text{ss}}, \tilde{b}^{\text{ss}}, m^{\text{ss}}$ and φ^{ss} replace $s^{\text{p}}, \tilde{b}^{\text{p}}, m_1$ and φ , and α ranges over $[0, 1]$. In Step 1, by Lemma 6(i), with $R^g(q) := s^g(q) - \tilde{b}^g(s^g(q))$ and $g := s^* - s^{\text{ss}}$, $M^g - \tilde{b}^g(M^g)$ has the law of $R^g(q_{(N)})$ and $R^{\text{ss}}(q) - R^*(q) = \int_0^q [g(w) - g(q)] k_q(w) dw$. In Step 2, $c_\alpha := m + \alpha \varphi^{\text{ss}} = (1 - \alpha)m + \alpha m^{\text{ss}}$ is continuous and non-decreasing in r at every $\alpha \in [0, 1]$ and increasing at every $\alpha \in [0, 1)$, m being increasing by $\zeta < 1$, so Lemma 4 applies at every $\alpha \in [0, 1]$, $\partial Q_\alpha(q)/\partial \alpha = -h_\alpha(Q_\alpha(q))$ at every $\alpha \in (0, 1)$, with $h_\alpha(t) := \mathbb{E}[\varphi^{\text{ss}}(r_i) | v_i - c_\alpha(r_i) = t]$, and $Q_\alpha(q)$ is Lipschitz in α on $[0, 1]$, since $|c_\alpha(r) - c_{\alpha'}(r)| \leq |\alpha - \alpha'| \max_{r'} |\varphi^{\text{ss}}(r')|$ at every r and α' moves every effective valuation, and hence every quantile, by at most that much, so that $g(q) = \int_0^1 h_\alpha(Q_\alpha(q)) d\alpha$. In Step 3, the log-submodularity of $\tilde{f}(t + c_\alpha(r), r)$ holds with c_α non-decreasing, both log-concavity inequalities being equalities where $c_\alpha(r) = c_\alpha(r')$. In Step 4, h_α is non-decreasing in t if φ^{ss} is non-increasing on $[\underline{r}, \bar{r}]$, and non-increasing in t if φ^{ss} is non-decreasing on $[\underline{r}, \bar{r}]$. By Step 1 of this proof, φ^{ss} is non-increasing on $[\underline{r}, \bar{r}]$ under assumption 3(a) and increasing under assumption 3(b), so g is non-decreasing under assumption 3(a) and non-increasing under assumption 3(b). Hence $R^{\text{ss}}(q) \leq R^*(q)$ at every $q \in (0, 1)$ under assumption 3(a) and $R^{\text{ss}}(q) \geq R^*(q)$ under assumption 3(b), and, $M^g - \tilde{b}^g(M^g)$ having the law of $R^g(q_{(N)})$, at every x under assumption 3(a)

$$\begin{aligned} \Pr(M^{\text{ss}} - \tilde{b}^{\text{ss}}(M^{\text{ss}}) \leq x) &= \Pr(R^{\text{ss}}(q_{(N)}) \leq x) \geq \Pr(R^*(q_{(N)}) \leq x) \\ &= \Pr(M^* - \tilde{b}^*(M^*) \leq x), \end{aligned}$$

and under assumption 3(b) the same chain holds with $\leq$ in place of $\geq$. For the strict inequalities in expectation stated after the proposition, $g(0^+) = m^{\text{ss}}(\bar{r}) - m(\bar{r}) = \varphi^{\text{ss}}(\bar{r})$ and $g(1^-) = \varphi^{\text{ss}}(\underline{r})$ by Lemma 4; by Step 1 of this proof, $\varphi^{\text{ss}}(\bar{r}) < \varphi^{\text{ss}}(\underline{r})$ under assumption 3(a), $r^{\text{ss}} < \bar{r}$ being maintained in section 3.1.4, and $\varphi^{\text{ss}}(\underline{r}) < \varphi^{\text{ss}}(\bar{r})$ under assumption 3(b); so g is not constant, and the bound of Step 5, respectively Step 6, of the proof of Proposition 4 gives the strict inequality in expectation. **Q.E.D.**

Proof of Proposition 9. Step 1 reduces the change in the interim surplus to the type's free market effective valuation $s = V^*(v_i, r_i)$ and the change $\varphi^{\text{ss}}(r_i)$ in his effective valuation, and signs the change in the interim surplus at the two ends $\underline{\varphi}$ and $\bar{\varphi}$ of the range of φ^{ss} ; Step 2 inverts the reduction into a threshold $r^\dagger(s)$; Step 3 reads parts (i)–(iii) off that threshold; Step 4 proves the footnote.

Write, in this proof, φ^{ss} for the distortion of Step 1 of the proof of Proposition 7, $\underline{\varphi} := \min_{r \in [\underline{r}, \bar{r}]} \varphi^{\text{ss}}(r)$, $\bar{\varphi} := \max_{r \in [\underline{r}, \bar{r}]} \varphi^{\text{ss}}(r)$, $\hat{r} := \max\{r^{\text{ss}}, \underline{r}\}$, $\underline{s} := V^*(\underline{v}, \bar{r})$, $\bar{s} := V^*(\bar{v}, \underline{r})$,

$\underline{s}^{\text{ss}} := V^{\text{ss}}(\underline{v}, \bar{r})$, and $A^g(x) := \int_{-\infty}^x F_V^g(t)^{N-1} dt$ for $g \in \{*, \text{ss}\}$, each F_V^g being 0 below its support. $r^{\text{ss}} < \bar{r}$ is maintained in section 3.1.4, so $\hat{r} < \bar{r}$.

Step 1: The reduction and the two ends. $V^{\text{ss}}(v_i, r_i) = V^*(v_i, r_i) - \varphi^{\text{ss}}(r_i)$, $u^*(v_i, r_i) = A^*(V^*(v_i, r_i))$ and $u^{\text{ss}}(v_i, r_i) = A^{\text{ss}}(V^{\text{ss}}(v_i, r_i))$, so

$$\Delta u^{\text{ss}}(v_i, r_i) = A^{\text{ss}}(V^*(v_i, r_i) - \varphi^{\text{ss}}(r_i)) - A^*(V^*(v_i, r_i)).$$

$V^*(v, r) = v - m(r)$ with m increasing, and $V^{\text{ss}}(v, r) = v - m^{\text{ss}}(r)$ with m^{ss} non-decreasing, so $\underline{s}$ and $\underline{s}^{\text{ss}} = \underline{s} - \varphi^{\text{ss}}(\bar{r})$ are the bottoms of the two supports, and, by Lemma 4, A^* is 0 on $(-\infty, \underline{s}]$ and increasing on $[\underline{s}, \infty)$, and A^{ss} is 0 on $(-\infty, \underline{s}^{\text{ss}}]$ and increasing on $[\underline{s}^{\text{ss}}, \infty)$. Since $\varphi^{\text{ss}}(r_j) - \underline{\varphi} \geq 0 \geq \varphi^{\text{ss}}(r_j) - \bar{\varphi}$ at every $r_j \in [\underline{r}, \bar{r}]$, at every t

$$\begin{aligned} F_V^{\text{ss}}(t - \underline{\varphi}) - F_V^*(t) &= \Pr(t < V^*(v_j, r_j) \leq t + \varphi^{\text{ss}}(r_j) - \underline{\varphi}) \geq 0, \\ F_V^*(t) - F_V^{\text{ss}}(t - \bar{\varphi}) &= \Pr(t - [\bar{\varphi} - \varphi^{\text{ss}}(r_j)] < V^*(v_j, r_j) \leq t) \geq 0. \end{aligned}$$

Both are positive at every t in some interval $(\underline{s}, \underline{s} + \varepsilon)$. Indeed, by Step 1 of the proof of Proposition 7, on (r^{ss}, ∞) the function φ^{ss} is continuous and differentiable except at r^* , with derivative $-\sigma^2[1 - r_S^2/(r + r_S)^2] < 0$ on (r^{ss}, r^*) , where $r > 0$, and a derivative of the sign of $\zeta - \zeta^{\text{ss}}(r)$ on (r^*, ∞) , which vanishes at most at one r , ζ^{ss} being increasing if $r_S > 0$ and equal to $1 > \zeta$ if $r_S = 0$; so some $\eta \in (0, \bar{r} - \hat{r})$ makes φ^{ss} strictly monotone on $[\bar{r} - \eta, \bar{r}]$, and then $\underline{\varphi} < \varphi^{\text{ss}}(r) < \bar{\varphi}$ at every $r \in (\bar{r} - \eta, \bar{r})$, $\varphi^{\text{ss}}(r)$ lying strictly between $\varphi^{\text{ss}}(\bar{r} - \eta)$ and $\varphi^{\text{ss}}(\bar{r})$. Take $\varepsilon \in (0, \bar{v} - \underline{v})$. At $t \in (\underline{s}, \underline{s} + \varepsilon)$, some $r_0 \in (\bar{r} - \eta, \bar{r})$ has $v_0 := t + m(r_0) \in (\underline{v}, \bar{v})$, since $t + m(r) \rightarrow t + m(\bar{r}) > \underline{v}$ as $r \rightarrow \bar{r}$ and $t + m(\bar{r}) < \bar{v}$; so (v_0, r_0) is an interior type with $V^*(v_0, r_0) = t$ and $\underline{\varphi} < \varphi^{\text{ss}}(r_0) < \bar{\varphi}$. By the continuity of m and φ^{ss} , the types near $(v_0 + \varepsilon', r_0)$ and those near $(v_0 - \varepsilon', r_0)$, for small $\varepsilon' > 0$, form open sets inside the first and the second event respectively, which therefore have positive probability, f_{vr} being positive. Hence, for every $s \in (\underline{s}, \bar{s}]$, substituting $w = t - \underline{\varphi}$, respectively $w = t - \bar{\varphi}$, in the integral defining A^{ss} ,

$$\begin{aligned} A^{\text{ss}}(s - \underline{\varphi}) - A^*(s) &= \int_{-\infty}^s [F_V^{\text{ss}}(t - \underline{\varphi})^{N-1} - F_V^*(t)^{N-1}] dt > 0, \\ A^*(s) - A^{\text{ss}}(s - \bar{\varphi}) &= \int_{-\infty}^s [F_V^*(t)^{N-1} - F_V^{\text{ss}}(t - \bar{\varphi})^{N-1}] dt > 0, \end{aligned}$$

each integrand being nonnegative and positive on $(\underline{s}, \min\{s, \underline{s} + \varepsilon\})$.

Step 2: The threshold. Write $(A^{\text{ss}})^{-1}$ for the inverse of the restriction of A^{ss} to $[\underline{s}^{\text{ss}}, \infty)$, which maps onto $[0, \infty)$, A^{ss} being continuous, 0 at $\underline{s}^{\text{ss}}$ and of slope 1 above the top of the support of $V^{\text{ss}}(v_i, r_i)$; $(A^{\text{ss}})^{-1}$ is continuous and increasing on $[0, \infty)$, and define, for every $s \in [\underline{s}, \bar{s}]$, $\chi(s) := s - (A^{\text{ss}})^{-1}(A^*(s))$, continuous since A^* and $(A^{\text{ss}})^{-1}$ are, so that $A^{\text{ss}}(s - \chi(s)) = A^*(s)$: $\chi(s)$ is the value of $\varphi^{\text{ss}}(r_i)$ that would leave a buyer with $V^*(v_i, r_i) = s$ exactly as well off under the policy as in the free market. At $s = \underline{s}$,

$A^*(\underline{s}) = 0$, so $\chi(\underline{s}) = \underline{s} - \underline{s}^{\text{ss}} = \varphi^{\text{ss}}(\bar{r})$. At $s \in (\underline{s}, \bar{s}]$, $A^*(s) > 0$, so $s - \chi(s) > \underline{s}^{\text{ss}}$, and $A^{\text{ss}}(x) < A^*(s)$ at $x < s - \chi(s)$ and $A^{\text{ss}}(x) > A^*(s)$ at $x > s - \chi(s)$; by Step 1, then, $s - \bar{\varphi} < s - \chi(s) < s - \underline{\varphi}$, that is, $\underline{\varphi} < \chi(s) < \bar{\varphi}$.

By Step 1 of the proof of Proposition 8, under assumption 3(a) φ^{ss} is non-increasing on $[\underline{r}, \bar{r}]$ and decreasing on $[\hat{r}, \bar{r}]$, and it is constant on $[\underline{r}, \hat{r}]$, equal to 0 on $[\underline{r}, r^{\text{ss}}]$; so $\bar{\varphi} = \varphi^{\text{ss}}(\hat{r})$ and $\underline{\varphi} = \varphi^{\text{ss}}(\bar{r})$. Under assumption 3(b), $r^{\text{ss}} \leq r^* \leq \underline{r}$ by Part (i) of the proof of Proposition 7, so $\hat{r} = \underline{r}$, and φ^{ss} is increasing on $[\underline{r}, \bar{r}]$, so $\underline{\varphi} = \varphi^{\text{ss}}(\underline{r})$ and $\bar{\varphi} = \varphi^{\text{ss}}(\bar{r})$. In either case the restriction of φ^{ss} to $[\hat{r}, \bar{r}]$ is a continuous and strictly monotone bijection onto $[\underline{\varphi}, \bar{\varphi}]$, which contains $\chi(s)$ at every $s \in [\underline{s}, \bar{s}]$; define $r^\dagger(s)$ as its inverse at $\chi(s)$. $r^\dagger(\cdot)$ is continuous, as the composition of two continuous maps; $r^\dagger(\underline{s}) = \bar{r}$, by $\chi(\underline{s}) = \varphi^{\text{ss}}(\bar{r})$; and $r^\dagger(s) \in (\hat{r}, \bar{r})$ at $s \in (\underline{s}, \bar{s}]$, by $\underline{\varphi} < \chi(s) < \bar{\varphi}$. Under assumption 3(a), for every $r_i \in [\underline{r}, \bar{r}]$ and $s \in [\underline{s}, \bar{s}]$,

$$\begin{aligned}\varphi^{\text{ss}}(r_i) &> \chi(s) &\iff r_i < r^\dagger(s), \\ \varphi^{\text{ss}}(r_i) &= \chi(s) &\iff r_i = r^\dagger(s), \\ \varphi^{\text{ss}}(r_i) &< \chi(s) &\iff r_i > r^\dagger(s),\end{aligned}$$

on $[\hat{r}, \bar{r}]$ because φ^{ss} is decreasing there, and on $[\underline{r}, \hat{r})$ because there $\varphi^{\text{ss}}(r_i) = \bar{\varphi} > \chi(s)$ and $r_i < \hat{r} < r^\dagger(s)$, using $\chi(\underline{s}) = \underline{\varphi} < \bar{\varphi}$ at $s = \underline{s}$. Under assumption 3(b), for every such r_i and s ,

$$\begin{aligned}\varphi^{\text{ss}}(r_i) &> \chi(s) &\iff r_i > r^\dagger(s), \\ \varphi^{\text{ss}}(r_i) &= \chi(s) &\iff r_i = r^\dagger(s), \\ \varphi^{\text{ss}}(r_i) &< \chi(s) &\iff r_i < r^\dagger(s),\end{aligned}$$

φ^{ss} being increasing on $[\underline{r}, \bar{r}] = [\hat{r}, \bar{r}]$.

Step 3: Parts (i)–(iii). Let $s := V^*(v_i, r_i)$. If $s > \underline{s}$, Steps 1 and 2 give $\Delta u^{\text{ss}}(v_i, r_i) = A^{\text{ss}}(s - \varphi^{\text{ss}}(r_i)) - A^{\text{ss}}(s - \chi(s))$, with $A^{\text{ss}}(x) < A^{\text{ss}}(s - \chi(s))$ at $x < s - \chi(s)$ and $A^{\text{ss}}(x) > A^{\text{ss}}(s - \chi(s))$ at $x > s - \chi(s)$, so

$$\begin{aligned}\Delta u^{\text{ss}}(v_i, r_i) &< 0 &\iff \varphi^{\text{ss}}(r_i) > \chi(s), \\ \Delta u^{\text{ss}}(v_i, r_i) &= 0 &\iff \varphi^{\text{ss}}(r_i) = \chi(s), \\ \Delta u^{\text{ss}}(v_i, r_i) &> 0 &\iff \varphi^{\text{ss}}(r_i) < \chi(s).\end{aligned}$$

If $s = \underline{s}$, then $(v_i, r_i) = (\underline{v}, \bar{r})$, m being increasing, and $\Delta u^{\text{ss}}(v_i, r_i) = A^{\text{ss}}(\underline{s}^{\text{ss}}) - A^*(\underline{s}) = 0$ and $\varphi^{\text{ss}}(r_i) = \varphi^{\text{ss}}(\bar{r}) = \chi(\underline{s})$, so the three equivalences hold there too. Combined with the equivalences of Step 2, they give parts (i)–(iii) under assumption 3(a) and under assumption 3(b).

Step 4: The footnote. The first paragraph of Step 2 and the first display of Step 3 use

neither assumption 3(a) nor 3(b). So, whether or not either holds, at $s = V^*(v_i, r_i) > \underline{s}$, $\Delta u^{\text{ss}}(v_i, r_i) = 0$ requires $\varphi^{\text{ss}}(r_i) = \chi(s) < \bar{\varphi} \leq 0$, hence $r_i > r^{\text{ss}}$. On $(r^{\text{ss}}, \bar{r}]$, φ^{ss} is strictly monotone on each of at most three intervals, split at r^* and at the root of $\zeta^{\text{ss}}(r) = \zeta$, by the derivatives of Step 1, so at most three r solve $\varphi^{\text{ss}}(r) = \chi(s)$. The closed set $\{(s, r) : \varphi^{\text{ss}}(r) = \chi(s)\}$ is therefore of Lebesgue measure zero, by Fubini's theorem, and so is its image under the Lipschitz map $(s, r) \mapsto (s + m(r), r)$, which contains every unaffected type other than $(\underline{v}, \bar{r})$. Let $r_i \leq r^{\text{ss}}$, whether or not assumption 3 holds. Then $\varphi^{\text{ss}}(r_i) = 0$ by Step 1 of the proof of Proposition 7, and $s = v_i - m(r_i) > \underline{v} - m(\bar{r}) = \underline{s}$, m being increasing and $r_i \leq r^{\text{ss}} < \bar{r}$, so

$$\Delta u^{\text{ss}}(v_i, r_i) = A^{\text{ss}}(s) - A^*(s) = - \int_{-\infty}^s \left[F_V^*(t)^{N-1} - F_V^{\text{ss}}(t)^{N-1} \right] dt < 0,$$

the integrand being nonnegative, by Step 2 of the proof of Proposition 7, and positive on the non-empty $(\underline{s}, s) \cap J$, with J from Step 3 of that proof. **Q.E.D.**

Proof of Lemma 2. Step 1 proves the two inequalities around $\Gamma(\hat{e})$ and when they are strict; Step 2 the two outer ones. Throughout, L is that of Step 1 of the proof of Proposition 10 at the belief in question, and the maximum over no buyers is $-\infty$.

Step 1: The inner inequalities. By part (a) of Step 2 of the proof of Proposition 10, $\Gamma(0) = \Delta_\omega - \mathbb{E}[\min\{L^+, \Delta_\omega\}]$ and $\Gamma(1) = \mathbb{E}[\min\{L^-, \Delta_\omega\}]$, and, at every $L \in [-\infty, +\infty]$, $\Delta_\omega - \min\{L^+, \Delta_\omega\} = \min\{(\Delta_\omega - L)^+, \Delta_\omega\}$, both sides being Δ_ω at $L \leq 0$ and $(\Delta_\omega - L)^+$ at $L \geq 0$, and $L^- = (-L)^+$. So, at either belief, $\Gamma(\hat{e}) = \mathbb{E}[\min\{((1 - \hat{e})\Delta_\omega - L)^+, \Delta_\omega\}]$. Its integrand equals Δ_ω where $L \leq -\hat{e}\Delta_\omega$, lies in $(0, \Delta_\omega)$ where $-\hat{e}\Delta_\omega < L < (1 - \hat{e})\Delta_\omega$, and equals 0 where $L \geq (1 - \hat{e})\Delta_\omega$. Hence $\Pr(L \leq -\hat{e}\Delta_\omega)\Delta_\omega \leq \Gamma(\hat{e}) \leq \Pr(L < (1 - \hat{e})\Delta_\omega)\Delta_\omega$, both inequalities strict if and only if $\Pr(-\hat{e}\Delta_\omega < L < (1 - \hat{e})\Delta_\omega) > 0$. Let $\underline{r} < r^*(\sigma_\epsilon^2) < \bar{r}$. By part (c) of Step 2 of the proof of Proposition 10, $\Pr(-\Delta_\omega < L < 0) > 0$ and $\Pr(0 < L < \Delta_\omega) > 0$, so $\Pr(-\hat{e}\Delta_\omega < L < (1 - \hat{e})\Delta_\omega) > 0$ at either belief, and $\Pr(L \leq -\hat{e}\Delta_\omega)\Delta_\omega < \Gamma(\hat{e}) < \Pr(L < (1 - \hat{e})\Delta_\omega)\Delta_\omega$. If $r^*(\sigma_\epsilon^2) \leq \underline{r}$, every buyer is contingent and $L = -\infty$; if $r^*(\sigma_\epsilon^2) \geq \bar{r}$, almost surely none is and $L = +\infty$; either way $\Pr(-\hat{e}\Delta_\omega < L < (1 - \hat{e})\Delta_\omega) = 0$ and $\Gamma(\hat{e}) = \Pr(L \leq -\hat{e}\Delta_\omega)\Delta_\omega = \Pr(L < (1 - \hat{e})\Delta_\omega)\Delta_\omega$.

Step 2: The outer inequalities. Each buyer is contingent independently with probability $\Pr(r_i \geq r^*(\sigma_\epsilon^2)) = 1 - F_r(r^*(\sigma_\epsilon^2))$, F_r being continuous, and $L = -\infty$ exactly when every buyer is contingent and $L = +\infty$ exactly when none is, so $[1 - F_r(r^*(\sigma_\epsilon^2))]^N = \Pr(L = -\infty) \leq \Pr(L \leq -\hat{e}\Delta_\omega)$ and $\Pr(L < (1 - \hat{e})\Delta_\omega) \leq \Pr(L < +\infty) = 1 - F_r(r^*(\sigma_\epsilon^2))^N$, the inequalities by inclusion of the events and the equalities by independence across buyers. **Q.E.D.**

Proof of Proposition 10. Step 1 reduces the seller's effort choice at a belief to the sign of $\Gamma(\hat{e}) - \theta$; Step 2 records four facts about $\Gamma(\cdot)$, used in the proofs of Lemmas 2 and

3; Step 3 assembles the equilibria.

Step 1: The seller's problem at a belief. Fix $\hat{e} \in \{0, 1\}$. Offers respond to the belief and not to her unobserved effort e : By Proposition 1 at $(\mu_{\hat{e}}, \sigma_{\hat{e}}^2)$, buyer i submits $b_i = \tilde{b}_i + d_i \omega_{\hat{e}}$, where $\tilde{b}_i := \tilde{b}^*(V^*(v_i, r_i | \hat{e}) | \hat{e})$ and $d_i := d^*(r_i | \hat{e}) = \mathbf{I}\{r_i \geq r^*(\sigma_{\hat{e}}^2)\}$. Her payoff from accepting offer i is $b_i - d_i \omega_e = \tilde{b}_i + d_i(\omega_{\hat{e}} - \omega_e)$, and retaining the object, worth 0, is dominated:

$$\begin{aligned} \tilde{b}_i + d_i(\omega_{\hat{e}} - \omega_e) &\geq V(1, \underline{v}, \bar{r} | \hat{e}) - \Delta_{\omega} \mathbf{I}\{\hat{e} = 1, e = 0\} \\ &\geq \min \{V(1, \underline{v}, \bar{r} | 0), V(1, \underline{v}, \bar{r} | 1) - \Delta_{\omega}\} \\ &= \min \left\{ V(1, \underline{v}, \bar{r} | 0), \underline{v} - \kappa - \mu_1 - \zeta \Delta_{\mu} - (1 - \zeta)^2 \bar{r} \sigma_1^2 - \zeta^2 r_S \sigma_0^2 \right\} \\ &= V(1, \underline{v}, \bar{r} | 0) > 0, \end{aligned}$$

the first inequality because $\tilde{b}_i \geq V^*(\underline{v}, \bar{r} | \hat{e}) \geq V(1, \underline{v}, \bar{r} | \hat{e})$ by Lemma 6(iii), $F_V^*(\cdot | \hat{e})$ being continuous and increasing on its support by Lemma 4, and because $\omega_0 - \omega_1 = \Delta_{\omega} > 0$, the second because the middle expression is $V(1, \underline{v}, \bar{r} | 0)$ at $\hat{e} = 0$ and at least $V(1, \underline{v}, \bar{r} | 1) - \Delta_{\omega}$ at $\hat{e} = 1$, the first equality expanding $\Delta_{\omega} = \zeta \Delta_{\mu} + \zeta^2 r_S \Delta_{\sigma^2}$ with $\sigma_1^2 + \Delta_{\sigma^2} = \sigma_0^2$, the second by $\zeta \Delta_{\mu} \leq \Delta_{\mu}$ and $\sigma_1^2 \leq \sigma_0^2$, and the last Lemma 5 at (μ_0, σ_0^2) . She therefore accepts the highest $\tilde{b}_i + d_i(\omega_{\hat{e}} - \omega_e)$, her expected payoff from effort e is $\mathbb{E}[\max_i(\tilde{b}_i + d_i(\omega_{\hat{e}} - \omega_e))] - \theta e$, and, $\Gamma(\hat{e})$ being the difference of the two expected maxima, as defined in section 3.2, $e = 1$ is a best response to $\hat{e}$ if and only if $\Gamma(\hat{e}) \geq \theta$ and $e = 0$ if and only if $\Gamma(\hat{e}) \leq \theta$.

Write, in this proof, w for the index of the highest $\tilde{b}_i$, almost surely unique since the types are independent across buyers, $F_V^*(\cdot | \hat{e})$ is continuous by Lemma 4 and $\tilde{b}^*(\cdot | \hat{e})$ is increasing by Lemma 6(iii), so that two buyers tie in $\tilde{b}_i$ with probability 0; $A := \max_{i: d_i=0} \tilde{b}_i$ and $B := \max_{i: d_i=1} \tilde{b}_i$, the maximum over no buyers being $-\infty$; and $L := A - B \in [-\infty, +\infty]$, which is $L(\sigma_{\hat{e}}^2)$ of section 3.2. Then $\tilde{b}_w = \max\{A, B\}$, $L^+ = \tilde{b}_w - B$, $L^- = \tilde{b}_w - A$, $\Pr(L = 0) = 0$ because $L = 0$ requires a tie between a waiving and a contingent buyer, $d_w = \mathbf{I}\{L < 0\}$ almost surely, and $\rho(\sigma_{\hat{e}}^2) = \Pr(L < 0) = \Pr(d_w = 1)$.

Step 2: Four facts about $\Gamma(\cdot)$. (a) At $\hat{e} = 0$ the two maxima in $\Gamma(0)$ are $\max_i(\tilde{b}_i + \Delta_{\omega} d_i) = \max\{A, B + \Delta_{\omega}\}$ and $\tilde{b}_w$; at $\hat{e} = 1$ they are $\tilde{b}_w$ and $\max_i(\tilde{b}_i - \Delta_{\omega} d_i) = \max\{A, B - \Delta_{\omega}\}$. At every $L \in [-\infty, +\infty]$,

$$\begin{aligned} \max\{A, B + \Delta_{\omega}\} - \max\{A, B\} &= \Delta_{\omega} \mathbf{I}\{L < 0\} + (\Delta_{\omega} - L)^+ \mathbf{I}\{L \geq 0\} \\ &= \Delta_{\omega} - \min\{L^+, \Delta_{\omega}\}, \\ \max\{A, B\} - \max\{A, B - \Delta_{\omega}\} &= \min\{-L, \Delta_{\omega}\} \mathbf{I}\{L < 0\} = \min\{L^-, \Delta_{\omega}\}, \end{aligned}$$

the first row reading $(B + \Delta_{\omega}) - B$ at $B > A$ and $\max\{A, B + \Delta_{\omega}\} - A$ at $A \geq B$, the second $B - \max\{A, B - \Delta_{\omega}\}$ at $B > A$ and $A - A$ at $A \geq B$. Taking expectations, with

$\Pr(L = 0) = 0$ and $\rho(\sigma_{\hat{e}}^2) = \Pr(L < 0)$,

$$\begin{aligned}\Gamma(0) &= \Delta_\omega - \mathbb{E}[\min\{L^+, \Delta_\omega\}] = \rho(\sigma_0^2)\Delta_\omega + \mathbb{E}[(\Delta_\omega - L)^+ \mathbf{I}\{L > 0\}], \\ \Gamma(1) &= \mathbb{E}[\min\{L^-, \Delta_\omega\}] = \rho(\sigma_1^2)\Delta_\omega - \mathbb{E}[(\Delta_\omega + L)^+ \mathbf{I}\{L < 0\}],\end{aligned}$$

the second equality of the second row because $\min\{-L, \Delta_\omega\} = \Delta_\omega - (\Delta_\omega + L)^+$ at $L < 0$; these are the forms stated in section 3.2.

(b) The integrands in (a) lie in $[0, \Delta_\omega]$, and the option terms $(\Delta_\omega - L)^+ \mathbf{I}\{L > 0\}$ and $(\Delta_\omega + L)^+ \mathbf{I}\{L < 0\}$ are nonnegative and vanish exactly off $\{0 < L < \Delta_\omega\}$ and off $\{-\Delta_\omega < L < 0\}$. Hence $0 \leq \Gamma(\hat{e}) \leq \Delta_\omega$, $\rho(\sigma_0^2)\Delta_\omega \leq \Gamma(0)$ and $\Gamma(1) \leq \rho(\sigma_1^2)\Delta_\omega$, the last two strict if and only if $\Pr(0 < L < \Delta_\omega) > 0$ at $\hat{e} = 0$ and $\Pr(-\Delta_\omega < L < 0) > 0$ at $\hat{e} = 1$, respectively.

(c) If $\underline{r} < r^*(\sigma_{\hat{e}}^2) < \bar{r}$, then $\Pr(-\Delta_\omega < L < 0) > 0$ and $\Pr(0 < L < \Delta_\omega) > 0$. Write, in this part, $V_j^* := V^*(v_j, r_j|\hat{e})$ and $c_{\hat{e}}(r) := \kappa + \mu_{\hat{e}} + (1 - \zeta)^2 r \sigma_{\hat{e}}^2 + \zeta^2 r_S \sigma_{\hat{e}}^2$, so that $V^*(v, r|\hat{e}) = v - c_{\hat{e}}(r)$ at $r \geq r^*(\sigma_{\hat{e}}^2)$ and $V^*(v, r|\hat{e}) = v - \mu_{\hat{e}} - r \sigma_{\hat{e}}^2$ at $r < r^*(\sigma_{\hat{e}}^2)$, with $\mu_{\hat{e}} + r^*(\sigma_{\hat{e}}^2) \sigma_{\hat{e}}^2 = c_{\hat{e}}(r^*(\sigma_{\hat{e}}^2))$ by the definition of $r^*(\sigma_{\hat{e}}^2)$. Put $s_0 := \bar{v} - c_{\hat{e}}(r^*(\sigma_{\hat{e}}^2))$; at every $\varepsilon \in (0, \bar{v} - \underline{v})$, $s_0 - \varepsilon > \underline{v} - c_{\hat{e}}(r^*(\sigma_{\hat{e}}^2)) > \underline{v} - c_{\hat{e}}(\bar{r}) = V^*(\underline{v}, \bar{r}|\hat{e})$, the bottom of the support of V_j^* , so $\tilde{b}^*(\cdot|\hat{e})$ is defined and continuous at $s_0 - \varepsilon$ and at s_0 by Lemma 6(iii), and some such ε has $\tilde{b}^*(s_0|\hat{e}) - \tilde{b}^*(s_0 - \varepsilon|\hat{e}) < \Delta_\omega$; fix it. Consider, with $E_3 := \bigcap_{j \geq 3} \{V_j^* < s_0 - \varepsilon\}$,

$$\begin{aligned}E_1 &:= \left\{d_1 = 1, s_0 - \frac{\varepsilon}{3} < V_1^* < s_0\right\} \cap \left\{d_2 = 0, s_0 - \varepsilon < V_2^* < s_0 - \frac{\varepsilon}{2}\right\} \cap E_3, \\ E_2 &:= \left\{d_1 = 0, s_0 - \frac{\varepsilon}{3} < V_1^* < s_0\right\} \cap \left\{d_2 = 1, s_0 - \varepsilon < V_2^* < s_0 - \frac{\varepsilon}{2}\right\} \cap E_3.\end{aligned}$$

Each has positive probability, the types being independent across buyers and f_{vr} positive: A contingent factor contains a neighbourhood of $(\bar{v} - \varepsilon/6, r)$ or of $(\bar{v} - 3\varepsilon/4, r)$ with r just above $r^*(\sigma_{\hat{e}}^2)$, and a waiving factor a neighbourhood of the same types with r just below it, both inside the type rectangle by the hypothesis and by $\varepsilon < \bar{v} - \underline{v}$, since $V^*(\cdot|\hat{e})$ is continuous with $V^*(\bar{v} - \varepsilon/6, r^*(\sigma_{\hat{e}}^2)|\hat{e}) = s_0 - \varepsilon/6$ and $V^*(\bar{v} - 3\varepsilon/4, r^*(\sigma_{\hat{e}}^2)|\hat{e}) = s_0 - 3\varepsilon/4$; and $\Pr(V_j^* < s_0 - \varepsilon) = F_V^*(s_0 - \varepsilon|\hat{e}) > 0$, $s_0 - \varepsilon$ lying above the bottom of the support of V_j^* , on which $F_V^*(\cdot|\hat{e})$ is increasing by Lemma 4. On E_1 , $\tilde{b}^*(\cdot|\hat{e})$ being increasing, $\tilde{b}_1 > \tilde{b}_2 > \max_{j \geq 3} \tilde{b}_j$, so $B = \tilde{b}_1$, $A = \tilde{b}_2$ and

$$-\Delta_\omega < \tilde{b}^*(s_0 - \varepsilon|\hat{e}) - \tilde{b}^*(s_0|\hat{e}) < L = \tilde{b}_2 - \tilde{b}_1 < 0;$$

on E_2 , likewise, $A = \tilde{b}_1$, $B = \tilde{b}_2$ and $0 < L = \tilde{b}_1 - \tilde{b}_2 < \tilde{b}^*(s_0|\hat{e}) - \tilde{b}^*(s_0 - \varepsilon|\hat{e}) < \Delta_\omega$.

(d) L and $\rho(\sigma_{\hat{e}}^2)$ depend on $\hat{e}$ only through $\sigma_{\hat{e}}^2$. Write, in this part, $\tilde{m}(r; \sigma^2) := \min\{r\sigma^2, \kappa + (1 - \zeta)^2 r \sigma^2 + \zeta^2 r_S \sigma^2\}$ and $\tilde{F}(\cdot; \sigma^2)$ for the CDF of $v_i - \tilde{m}(r_i; \sigma^2)$, so that $V^*(v, r|\hat{e}) = v - \mu_{\hat{e}} - \tilde{m}(r; \sigma_{\hat{e}}^2)$ and $F_V^*(s|\hat{e}) = \tilde{F}(s + \mu_{\hat{e}}; \sigma_{\hat{e}}^2)$. Substituting $u = t + \mu_{\hat{e}}$ in

the definition of $\tilde{b}^*(\cdot|\hat{e})$, with $V^*(\underline{v}, \bar{r}|\hat{e}) + \mu_{\hat{e}} = \underline{v} - \tilde{m}(\bar{r}; \sigma_{\hat{e}}^2)$,

$$\tilde{b}^*(s|\hat{e}) + \mu_{\hat{e}} = s + \mu_{\hat{e}} - \frac{\int_{\underline{v}-\tilde{m}(\bar{r}; \sigma_{\hat{e}}^2)}^{s+\mu_{\hat{e}}} \tilde{F}(u; \sigma_{\hat{e}}^2)^{N-1} du}{\tilde{F}(s + \mu_{\hat{e}}; \sigma_{\hat{e}}^2)^{N-1}},$$

a function of $s + \mu_{\hat{e}}$ and $\sigma_{\hat{e}}^2$ alone. So $\tilde{b}_i + \mu_{\hat{e}}$ is a function of $(v_i, r_i, \sigma_{\hat{e}}^2)$, as is $d_i = \mathbf{I}\{r_i \geq r^*(\sigma_{\hat{e}}^2)\}$; $\mu_{\hat{e}}$ cancels from $L = A - B$, and $\rho(\sigma_{\hat{e}}^2) = \Pr(L < 0)$ follows.

Step 3: Equilibria. A pure effort profile is an equilibrium if and only if the belief is correct and the effort is a best response to it. By Step 1, $\hat{e} = e = 1$ is an equilibrium if and only if $\Gamma(1) \geq \theta$, and $\hat{e} = e = 0$ if and only if $\Gamma(0) \leq \theta$. **Q.E.D.**

Proof of Lemma 3. Step 1 reduces the claim to a comparison of $\rho(\sigma_1^2)$ with $\rho(\sigma_0^2)$, the probabilities of Step 1 of the proof of Proposition 10, and settles the case $r^*(\sigma_1^2) \geq \bar{r}$; Step 2 makes the comparison under the other two conditions; Step 3 computes the gap under condition (i).

Step 1: The reduction. If $r^*(\sigma_1^2) \geq \bar{r}$, a buyer is contingent at $\hat{e} = 1$ only if $r_i \geq \bar{r}$, an event of probability zero, so $\rho(\sigma_1^2) = 0$ and $0 \leq \Gamma(1) \leq \rho(\sigma_1^2)\Delta_{\omega} = 0$ by part (b) of Step 2 of the proof of Proposition 10; and, since $r^*(\sigma_0^2) < \bar{r}$ and $f_r > 0$, every buyer has $r_j > r^*(\sigma_0^2)$ with positive probability, the believed winner is then contingent, and $\Gamma(0) \geq \rho(\sigma_0^2)\Delta_{\omega} > 0 = \Gamma(1)$ by its part (b). If $r^*(\sigma_1^2) < \bar{r}$, then $r^*(\sigma_1^2) \in (\underline{r}, \bar{r})$, parts (b) and (c) give $\Gamma(1) < \rho(\sigma_1^2)\Delta_{\omega}$, and whenever $\rho(\sigma_1^2) \leq \rho(\sigma_0^2)$,

$$\Gamma(1) < \rho(\sigma_1^2)\Delta_{\omega} \leq \rho(\sigma_0^2)\Delta_{\omega} \leq \Gamma(0),$$

the last inequality by part (b).

Step 2: The two probabilities. Write $w_{\hat{e}}$ for the index of the highest $V^*(v_i, r_i|\hat{e})$, almost surely unique, so that $\rho(\sigma_{\hat{e}}^2) = \Pr(d^*(r_{w_{\hat{e}}}|\hat{e}) = 1)$, and recall $d^*(r|\hat{e}) = \mathbf{I}\{r \geq r^*(\sigma_{\hat{e}}^2)\}$. If $\Delta_{\sigma^2} = 0$, then $V^*(v, r|0) + \mu_0 = V^*(v, r|1) + \mu_1$ and $d^*(r|0) = d^*(r|1)$ at every (v, r) , so $w_0 = w_1$ and $\rho(\sigma_0^2) = \rho(\sigma_1^2)$. Suppose instead $(1 - \zeta)^2\bar{r} + \zeta^2 r_S \leq \underline{r}$, and note $r^*(\sigma_1^2) - r^*(\sigma_0^2) = \kappa\Delta_{\sigma^2}/[\zeta(2 - \zeta)\sigma_0^2\sigma_1^2] \geq 0$. Fix a realization at which w_0 and w_1 are unique and $r_{w_1} \geq r^*(\sigma_1^2)$, suppose $r_{w_0} < r^*(\sigma_0^2)$, and write $i := w_1$ and $j := w_0$. Then $r_j < r^*(\sigma_0^2) \leq r^*(\sigma_1^2) \leq r_i$, so $i \neq j$, buyer i is contingent and buyer j waives at both beliefs, and

$$\begin{aligned} V^*(v_i, r_i|0) - V^*(v_j, r_j|0) &= V^*(v_i, r_i|1) - V^*(v_j, r_j|1) - \left[(1 - \zeta)^2 r_i + \zeta^2 r_S - r_j\right] \Delta_{\sigma^2} \\ &> - \left[(1 - \zeta)^2 r_i + \zeta^2 r_S - r_j\right] \Delta_{\sigma^2} \\ &\geq - \left[(1 - \zeta)^2 \bar{r} + \zeta^2 r_S - \underline{r}\right] \Delta_{\sigma^2} \geq 0, \end{aligned}$$

the equality because $V^*(v_i, r_i|\hat{e}) = V(1, v_i, r_i|\hat{e})$ and $V^*(v_j, r_j|\hat{e}) = V(0, v_j, r_j|\hat{e})$ at both beliefs, the mean gap Δ_{μ} entering both and cancelling, the first inequality because i

has the highest $V^*(\cdot|1)$, the second by $r_i \leq \bar{r}$, $r_j \geq \underline{r}$ and $\Delta_{\sigma^2} \geq 0$, and the last by the condition. This contradicts $j = w_0$, so $d^*(r_{w_1}|1) = 1$ implies $d^*(r_{w_0}|0) = 1$ almost surely, and $\rho(\sigma_1^2) \leq \rho(\sigma_0^2)$. *Step 3: The gap when $\Delta_{\sigma^2} = 0$.* By part (d) of Step 2 of the proof of Proposition 10, L and ρ depend on the belief only through $\sigma_{\hat{e}}^2$, so at $\sigma_1^2 = \sigma_0^2$ they coincide across beliefs; write L and ρ . With $\Delta_\omega = \zeta\Delta_\mu$,

$$\Gamma(0) - \Gamma(1) = \mathbb{E} \left[(\zeta\Delta_\mu - L)^+ \mathbf{I}\{L > 0\} + (\zeta\Delta_\mu + L)^+ \mathbf{I}\{L < 0\} \right] = \mathbb{E} \left[(\zeta\Delta_\mu - |L|)^+ \right],$$

the first equality subtracting the two forms of part (a) of Step 2 of that proof, the terms $\rho\zeta\Delta_\mu$ cancelling, and the second because $\{L = 0\}$ has probability zero. If $\underline{r} < r^*(\sigma_0^2) < \bar{r}$, part (c) of Step 2 of that proof gives $\Pr(-\zeta\Delta_\mu < L < 0) > 0$ and $\Pr(0 < L < \zeta\Delta_\mu) > 0$, so the gap is positive. **Q.E.D.**

Proof of Proposition 11. Fix $\hat{e} \in \{0, 1\}$. Assumption 2 holds at $(\mu_{\hat{e}}, \sigma_{\hat{e}}^2)$, so by Proposition 2 at those moments buyer i submits $b_i = \tilde{b}^p(V(1, v_i, r_i|\hat{e})|\hat{e}) + \omega_{\hat{e}}$, free of her effort e , and inspects if he wins. Her payoff from accepting offer i is $b_i - \omega_e$, and retaining the object, worth 0, is dominated:

$$b_i - \omega_e \geq V(1, \underline{v}, \bar{r}|\hat{e}) - \Delta_\omega \mathbf{I}\{\hat{e} = 1, e = 0\} \geq V(1, \underline{v}, \bar{r}|0) > 0,$$

the first inequality by Lemma 6(iii), $F_V^p(\cdot|\hat{e})$ being continuous and increasing on its support by Lemma 4, and by $\omega_0 - \omega_1 = \Delta_\omega > 0$, the others as in Step 1 of the proof of Proposition 10. She therefore accepts the highest b_i whatever e , her expected payoff from effort e is $\mathbb{E}[\max_i b_i] - \omega_e - \theta e$, and her gain from maintaining is $\Delta_\omega - \theta$ at either belief. A pure effort profile is an equilibrium if and only if the belief is correct and the effort is a best response to it: $\hat{e} = e = 1$ is one if and only if $\Delta_\omega \geq \theta$, and $\hat{e} = e = 0$ if and only if $\Delta_\omega \leq \theta$; at $\Delta_\omega \neq \theta$ exactly one of the two holds. **Q.E.D.**

Proof of Proposition 12. Read F_V^* , $\tilde{b}^*$ and s^* at (μ_0, σ_0^2) and F_V^p , $\tilde{b}^p$ and s^p at (μ_1, σ_1^2) throughout, their moments suppressed, s^g being the inverse of F_V^g on its support. Step 1 derives the closed form of ψ and brackets the change in realized total surplus by $\psi(\underline{r})$ and $\psi(\bar{r})$; Step 2 orders the two regimes' effective valuations rank by rank and Step 3 their bids at a common rank; Step 4 couples the two auctions on the ranks and reads off the distributional conclusions; Step 5 assembles the proposition, proves its footnote and writes $\psi(r)$ in the primitives; Step 6 reads the first condition against the maintained assumption $\theta \in [\Gamma(0), \Delta_\omega]$.

Step 1: The boost and realized total surplus. Write, in this proof, $m(r|0) := \min\{\mu_0 + r\sigma_0^2, \kappa + \mu_0 + r(1 - \zeta)^2\sigma_0^2 + \zeta^2 r_s \sigma_0^2\}$ and $c_1(r) := \kappa + \mu_1 + r(1 - \zeta)^2\sigma_1^2 + \zeta^2 r_s \sigma_1^2$ for the cost sides in the free market and under the no-waiver policy, so that $V^*(v, r|0) = v - m(r|0)$, $V(1, v, r|1) = v - c_1(r)$ and $\psi = m(\cdot|0) - c_1$. The second branch of $m(\cdot|0)$ exceeds the

first by $\kappa + \zeta^2 r_S \sigma_0^2 - \zeta(2 - \zeta) \sigma_0^2 r = \zeta(2 - \zeta) \sigma_0^2 [r^*(\sigma_0^2) - r]$, by the definition of $r^*(\sigma_0^2)$, so, with $(1 - \zeta)^2 + \zeta(2 - \zeta) = 1$ and $\sigma_1^2 = \sigma_0^2 - \Delta_{\sigma^2}$,

$$\begin{aligned}\psi(r) &= \Delta_\mu + \left[(1 - \zeta)^2 r + \zeta^2 r_S\right] \Delta_{\sigma^2} - \varphi(r|0), \\ \psi'(r) &= (1 - \zeta)^2 \Delta_{\sigma^2} + \zeta(2 - \zeta) \sigma_0^2 \mathbf{I}\{r < r^*(\sigma_0^2)\} \geq 0 \quad \text{at } r \neq r^*(\sigma_0^2) : \end{aligned}$$

ψ is continuous and non-decreasing, increasing on $[\underline{r}, r^*(\sigma_0^2)] \cap [\underline{r}, \bar{r}]$ and, if $\Delta_{\sigma^2} > 0$, on all of $[\underline{r}, \bar{r}]$. Since $V(1, v_j, r_j|1) = V^*(v_j, r_j|0) + \psi(r_j)$ at every j , with i a buyer attaining $M_{\text{mh}}^* = \max_j V^*(v_j, r_j|0)$,

$$\begin{aligned}\psi(\underline{r}) \leq \psi(r_i) &\leq M_{\text{mh}}^{\text{P}} - M_{\text{mh}}^* = \max_{j \in \mathcal{N}} [V^*(v_j, r_j|0) + \psi(r_j)] - V^*(v_i, r_i|0) \\ &\leq \max_{j \in \mathcal{N}} \psi(r_j) \leq \psi(\bar{r}), \end{aligned}$$

the second inequality reading the maximum at $j = i$, the third because $V^*(v_j, r_j|0) \leq V^*(v_i, r_i|0)$ at every j , and the outer bounds because ψ is non-decreasing. Realized total surplus is M_{mh}^* in the free market and $M_{\text{mh}}^{\text{P}} - \theta$ under the no-waiver policy, so $\psi(\underline{r}) - \theta \leq (M_{\text{mh}}^{\text{P}} - \theta) - M_{\text{mh}}^* \leq \psi(\bar{r}) - \theta$ at every realization.

Step 2: The effective valuations rank by rank. By Lemma 4 at the cost sides $m(\cdot|0)$ and c_1 , both continuous and increasing, each F_V^{g} is continuous, increasing on its support and 0 below it, and $s^{\text{g}}(q)$ solves $F_V^{\text{g}}(t) = q$ at every $q \in (0, 1]$. Since $V(1, v_j, r_j|1) = V^*(v_j, r_j|0) + \psi(r_j)$ and $\psi(\underline{r}) \leq \psi(r_j) \leq \psi(\bar{r})$, at every t ,

$$F_V^*(t - \psi(\bar{r})) \leq F_V^{\text{P}}(t) = \Pr(V^*(v_j, r_j|0) \leq t - \psi(r_j)) \leq F_V^*(t - \psi(\underline{r})).$$

Read at $t = s^{\text{P}}(q)$ and at $t = s^*(q) + \psi(\bar{r})$, for every $q \in (0, 1]$,

$$\begin{aligned}F_V^*(s^*(q)) = q = F_V^{\text{P}}(s^{\text{P}}(q)) &\leq F_V^*(s^{\text{P}}(q) - \psi(\underline{r})) \implies s^*(q) \leq s^{\text{P}}(q) - \psi(\underline{r}), \\ F_V^{\text{P}}(s^{\text{P}}(q)) = q = F_V^*(s^*(q)) &\leq F_V^{\text{P}}(s^*(q) + \psi(\bar{r})) \implies s^{\text{P}}(q) \leq s^*(q) + \psi(\bar{r}), \end{aligned}$$

each implication because $F_V^{\text{g}}(y) < F_V^{\text{g}}(s^{\text{g}}(q)) = q$ at every $y < s^{\text{g}}(q)$, F_V^{g} being increasing on its support and 0 below it. Hence $\psi(\underline{r}) \leq s^{\text{P}}(q) - s^*(q) \leq \psi(\bar{r})$ at every $q \in (0, 1]$.

Step 3: The bids at a common rank. By Lemma 6(i) with $w = qz$, in each regime $\tilde{b}^{\text{g}}(s^{\text{g}}(q)) = \int_0^1 s^{\text{g}}(qz)(N - 1)z^{N-2}dz$ at every $q \in (0, 1]$, and $\int_0^1 (N - 1)z^{N-2}dz = 1$, so by Step 2

$$\psi(\underline{r}) \leq \tilde{b}^{\text{P}}(s^{\text{P}}(q)) - \tilde{b}^*(s^*(q)) = \int_0^1 [s^{\text{P}}(qz) - s^*(qz)](N - 1)z^{N-2}dz \leq \psi(\bar{r}).$$

Step 4: Seller surplus, by coupling on the ranks. Let $q_1, \dots, q_N$ be i.i.d. $\text{U}[0, 1]$ and $q_{(N)}$ their maximum. In regime g the effective valuations have the law of $(s^{\text{g}}(q_j))_{j \in \mathcal{N}}$,

F_V^g being continuous, and s^g is increasing, so M_{mh}^g has the law of $s^g(q_{(N)})$ and seller surplus, $\tilde{b}^*(M_{mh}^*|0)$ in the free market and $\tilde{b}^p(M_{mh}^p|1) - \theta$ under the no-waiver policy, the law of $\tilde{b}^*(s^*(q_{(N)}))$ and of $\tilde{b}^p(s^p(q_{(N)})) - \theta$. By Step 3 at $q = q_{(N)}$, positive almost surely, $\{\tilde{b}^p(s^p(q_{(N)})) - \theta \leq x\} \subseteq \{\tilde{b}^*(s^*(q_{(N)})) \leq x\}$ almost surely at every x if $\psi(\underline{r}) > \theta$, and $\supseteq$ if $\psi(\bar{r}) < \theta$; so, at every x ,

$$\begin{aligned}\psi(\underline{r}) > \theta &\implies \Pr(\tilde{b}^p(M_{mh}^p|1) - \theta \leq x) \leq \Pr(\tilde{b}^*(M_{mh}^*|0) \leq x), \\ \psi(\bar{r}) < \theta &\implies \Pr(\tilde{b}^p(M_{mh}^p|1) - \theta \leq x) \geq \Pr(\tilde{b}^*(M_{mh}^*|0) \leq x).\end{aligned}$$

Step 5: The proposition, its footnote, and $\psi(\underline{r})$. By Step 1, $\theta < \psi(\underline{r})$ gives $M_{mh}^p - \theta > M_{mh}^*$ and $\theta > \psi(\bar{r})$ gives $M_{mh}^p - \theta < M_{mh}^*$ at every realization; by Step 4 they give the two comparisons of seller surplus. By Step 1, at every $r \geq r^*(\sigma_0^2)$,

$$\begin{aligned}\psi(r) - \theta &= \Delta_\mu - \theta + [(1 - \zeta)^2 r + \zeta^2 r_S] \Delta_{\sigma^2} \\ &> (1 - \zeta) \Delta_\mu + (1 - \zeta)^2 r \Delta_{\sigma^2} \geq (1 - \zeta) \Delta_\mu > 0,\end{aligned}$$

the inequalities by $\theta < \Delta_\omega = \zeta \Delta_\mu + \zeta^2 r_S \Delta_{\sigma^2}$, the assumption of section 3.2.3, by $r \geq 0$ and $\Delta_{\sigma^2} \geq 0$, and by $\zeta < 1$ and $\Delta_\mu > 0$; so $\theta > \psi(\bar{r})$ forces $\bar{r} < r^*(\sigma_0^2)$, the footnote to the statement. At $\underline{r} < r^*(\sigma_0^2)$, the assumption of section 3.2.3, Step 1 gives

$$\begin{aligned}\psi(\underline{r}) &= \Delta_\mu + [(1 - \zeta)^2 \underline{r} + \zeta^2 r_S] \Delta_{\sigma^2} - \zeta(2 - \zeta) \sigma_0^2 [r^*(\sigma_0^2) - \underline{r}] \\ &= \Delta_\mu + \underline{r} [\sigma_0^2 - (1 - \zeta)^2 \sigma_1^2] - \zeta^2 r_S \sigma_1^2 - \kappa \\ &= \Delta_\mu + \underline{r} \Delta_{\sigma^2} - \zeta(2 - \zeta) \sigma_1^2 [r^*(\sigma_1^2) - \underline{r}],\end{aligned}$$

the second equality by $\zeta(2 - \zeta) \sigma_0^2 r^*(\sigma_0^2) = \kappa + \zeta^2 r_S \sigma_0^2$, $(1 - \zeta)^2 \Delta_{\sigma^2} + \zeta(2 - \zeta) \sigma_0^2 = \sigma_0^2 - (1 - \zeta)^2 \sigma_1^2$ and $\zeta^2 r_S (\Delta_{\sigma^2} - \sigma_0^2) = -\zeta^2 r_S \sigma_1^2$, and the third by $\zeta(2 - \zeta) \sigma_1^2 r^*(\sigma_1^2) = \kappa + \zeta^2 r_S \sigma_1^2$ and $\Delta_{\sigma^2} + \zeta(2 - \zeta) \sigma_1^2 = \sigma_0^2 - (1 - \zeta)^2 \sigma_1^2$: the gain from maintenance at $\underline{r}$, the first row of the decomposition of ψ since $\underline{r} < r^*(\sigma_0^2)$, less the risk-sharing distortion there, $\underline{r} < r^*(\sigma_0^2) \leq r^*(\sigma_1^2)$.

Step 6: The first condition under the maintained assumption. The costs section 3.2.3 admits at which the first condition holds are $\Theta := \{\theta > 0 : \Gamma(0) \leq \theta < \min\{\psi(\underline{r}), \Delta_\omega\}\}$. Lemma 2 at $\hat{e} = 0$ gives $\Gamma(0) \leq [1 - F_r(r^*(\sigma_0^2))^N] \Delta_\omega < \Delta_\omega$, the last because $r^*(\sigma_0^2) > \underline{r}$ and $f_r > 0$, so

$$\Theta \neq \emptyset \iff \Gamma(0) < \psi(\underline{r}), \quad \Theta = \{\theta > 0 : \Gamma(0) \leq \theta < \Delta_\omega\} \iff \psi(\underline{r}) \geq \Delta_\omega,$$

the first because Θ is the interval $[\Gamma(0), \min\{\psi(\underline{r}), \Delta_\omega\})$ less $\{0\}$ and $\Gamma(0) < \Delta_\omega$, and the second because $\min\{\psi(\underline{r}), \Delta_\omega\} = \Delta_\omega$ at $\psi(\underline{r}) \geq \Delta_\omega$, while at $\psi(\underline{r}) < \Delta_\omega$ every positive $\theta \in [\max\{\Gamma(0), \psi(\underline{r})\}, \Delta_\omega)$, a non-empty set since $0 \leq \Gamma(0) < \Delta_\omega$, is admissible

and outside Θ . By Step 1 at $\underline{r} < r^*(\sigma_0^2)$ and $\Delta_\omega = \zeta\Delta_\mu + \zeta^2 r_S \Delta_{\sigma^2}$, $\psi(\underline{r}) - \Delta_\omega = (1 - \zeta)\Delta_\mu + (1 - \zeta)^2 \underline{r} \Delta_{\sigma^2} - \varphi(\underline{r}|0)$. **Q.E.D.**

Proof of Proposition 13. The proof mirrors the proof of Proposition 4, with the boost ψ , non-decreasing, in place of the distortion φ , non-increasing. Step 1 writes aggregate buyer surplus in each regime through the winner's rank; Step 2 writes the rank gap as an integral of the boost's posterior mean along a homotopy between the two cost sides; Step 3 shows the gap is non-increasing and not constant; Step 4 concludes. Read F_V^* , $\tilde{b}^*$ and s^* at (μ_0, σ_0^2) and F_V^p , $\tilde{b}^p$ and s^p at (μ_1, σ_1^2) , their moments suppressed, s^g being the inverse of F_V^g on its support, and keep $m(\cdot|0)$ and c_1 from Step 1 of the proof of Proposition 12, so that $V^*(v_i, r_i|0) = v_i - m(r_i|0)$, $V(1, v_i, r_i|1) = v_i - c_1(r_i)$ and $\psi = m(\cdot|0) - c_1$, with $m(\cdot|0)$ and c_1 continuous and increasing, of slopes σ_0^2 or $(1 - \zeta)^2 \sigma_0^2$ and $(1 - \zeta)^2 \sigma_1^2$.

Step 1: Aggregate buyer surplus through the winner's rank. By Lemma 4 at $m(\cdot|0)$ and at c_1 , each F_V^g is continuous and increasing on its support, so Step 1 of the proof of Proposition 4 holds in each regime: With $q_{(N)}$ the largest of N i.i.d. $U[0, 1]$ ranks, $k_q(w) = (N - 1)w^{N-2}/q^{N-1}$ and $R^g(q) := s^g(q) - \tilde{b}^g(s^g(q))$, $M_{mh}^g - \tilde{b}^g(M_{mh}^g)$ has the law of $R^g(q_{(N)})$, and, by Lemma 6(i), with $g_{mh}(q) := s^p(q) - s^*(q)$,

$$R^p(q) - R^*(q) = \int_0^q [g_{mh}(q) - g_{mh}(w)] k_q(w) dw.$$

Step 2: The rank gap through the boost's posterior mean. Write, in this proof, $c_\alpha := (1 - \alpha)m(\cdot|0) + \alpha c_1 = m(\cdot|0) - \alpha\psi$ for $\alpha \in (-\varepsilon, 1 + \varepsilon)$, $\varepsilon := (1 - \zeta)^2 \sigma_1^2 / \sigma_0^2$, continuous and increasing: Off $r^*(\sigma_0^2)$ its slope $(1 - \alpha)m'(\cdot|0) + \alpha c_1'$ is a convex combination of positive slopes at $\alpha \in [0, 1]$, at least $m'(\cdot|0) > 0$ at $\alpha < 0$, and at least $(1 - \zeta)^2 \sigma_1^2 - (\alpha - 1)\sigma_0^2 > 0$ at $1 < \alpha < 1 + \varepsilon$, since $c_1' \leq m'(\cdot|0) \leq \sigma_0^2$ by $\sigma_1^2 \leq \sigma_0^2$; $c_0 = m(\cdot|0)$; $V_\alpha := v_i - c_\alpha(r_i)$, with CDF F_α , density f_α and quantile $Q_\alpha(q)$ at $q \in (0, 1)$, so that $Q_0 = s^*$ and $Q_1 = s^p$; $\bar{f}(x, r) := f_{vr}(x, r)\mathbf{I}\{\underline{v} \leq x \leq \bar{v}\}$; and $h_\alpha(t) := \mathbb{E}[\psi(r_i)|V_\alpha = t]$. Step 2 of the proof of Proposition 4 uses of the cost side only that it is continuous and increasing, of the distortion only that it is bounded, and of f_{vr} only that it is bounded and positive with $f_{vr}(\cdot, r)$ continuous, so it holds at c_α with $\partial c_\alpha / \partial \alpha = -\psi$ in place of φ :

$$f_\alpha(t) = \int_{\underline{r}}^{\bar{r}} \bar{f}(t + c_\alpha(r), r) dr, \quad f_\alpha(t)h_\alpha(t) = \int_{\underline{r}}^{\bar{r}} \psi(r)\bar{f}(t + c_\alpha(r), r) dr,$$

$$\frac{\partial F_\alpha(t)}{\partial \alpha} = -f_\alpha(t)h_\alpha(t),$$

and, by the implicit function theorem at $F_\alpha(Q_\alpha(q)) = q$, where $f_\alpha(Q_\alpha(q)) > 0$,

$$\frac{\partial Q_\alpha(q)}{\partial \alpha} = h_\alpha(Q_\alpha(q)), \quad g_{mh}(q) = Q_1(q) - Q_0(q) = \int_0^1 h_\alpha(Q_\alpha(q)) d\alpha.$$

Step 3: g_{mh} is non-increasing, from $\psi(\bar{r})$ to $\psi(\underline{r})$. Steps 3 and 4 of the proof of

Proposition 4 use of the cost side only that it is continuous and increasing, so they hold at c_α : Fix $\alpha \in [0, 1]$ and $t < t'$ in the interior of the support of V_α , and write $a_t(r) := \bar{f}(t + c_\alpha(r), r)$; at $r < r'$, $a_t(r)a_{t'}(r') \leq a_t(r')a_{t'}(r)$, and

$$f_\alpha(t)f_\alpha(t') [h_\alpha(t') - h_\alpha(t)] = \frac{1}{2} \iint_{[\underline{r}, \bar{r}]^2} [\psi(r') - \psi(r)] [a_t(r)a_{t'}(r') - a_t(r')a_{t'}(r)] dr dr' \leq 0,$$

the inequality because at $r < r'$ the first bracket is nonnegative, ψ being non-decreasing, and the second is nonpositive; at $r > r'$ both signs reverse; and at $r = r'$ the integrand is 0. So h_α is non-increasing on that interior, Q_α is increasing, and by Step 2 g_{mh} is non-increasing on $(0, 1)$. $Q_\alpha(q)$ tends to the bottom $\underline{v} - c_\alpha(\bar{r})$ of the support of V_α as $q \rightarrow 0$ and to its top $\bar{v} - c_\alpha(\underline{r})$ as $q \rightarrow 1$, by Lemma 4, so

$$g_{\text{mh}}(0^+) = m(\bar{r}|0) - c_1(\bar{r}) = \psi(\bar{r}) > \psi(\underline{r}) = m(\underline{r}|0) - c_1(\underline{r}) = g_{\text{mh}}(1^-),$$

the inequality because ψ is non-decreasing, and increasing on $[\underline{r}, \min\{r^*(\sigma_0^2), \bar{r}\}]$, by Step 1 of the proof of Proposition 12, that interval being non-degenerate by $r^*(\sigma_0^2) > \underline{r}$, the assumption of section 3.2.3.

Step 4: The no-waiver policy FOSD-decreases aggregate buyer surplus, strictly in expectation. By Step 3, $g_{\text{mh}}(w) \geq g_{\text{mh}}(q)$ at $w \leq q$, so Step 1's display gives $R^p(q) \leq R^*(q)$ at every $q \in (0, 1)$ and, at every x ,

$$\begin{aligned} \Pr(M_{\text{mh}}^p - \tilde{b}^p(M_{\text{mh}}^p) \leq x) &= \Pr(R^p(q_{(N)}) \leq x) \geq \Pr(R^*(q_{(N)}) \leq x) \\ &= \Pr(M_{\text{mh}}^* - \tilde{b}^*(M_{\text{mh}}^*) \leq x). \end{aligned}$$

For the footnote, g_{mh} is non-increasing and not constant by Step 3, so some $0 < w_1 < q_1 < 1$ have $g_{\text{mh}}(w_1) > g_{\text{mh}}(q_1)$, and, as in Step 5 of the proof of Proposition 4,

$$\begin{aligned} \mathbb{E}[M_{\text{mh}}^* - \tilde{b}^*(M_{\text{mh}}^*)] - \mathbb{E}[M_{\text{mh}}^p - \tilde{b}^p(M_{\text{mh}}^p)] &= \int_0^1 \int_0^q [g_{\text{mh}}(w) - g_{\text{mh}}(q)] k_q(w) dw N q^{N-1} dq \\ &\geq [g_{\text{mh}}(w_1) - g_{\text{mh}}(q_1)] N w_1^{N-1} (1 - q_1) > 0. \end{aligned}$$

Q.E.D.

Proof of Proposition 14. Step 1 writes the change in a buyer's interim surplus through his free-market effective valuation and his boost; Step 2 signs it at the two extreme boosts; Step 3 inverts the boost at which it vanishes into the threshold $r^\dagger(\cdot)$ and reads parts (i)–(iii) off it. Read F_V^* at (μ_0, σ_0^2) and F_V^p at (μ_1, σ_1^2) , their moments suppressed, keep $m(\cdot|0)$ and c_1 from Step 1 of the proof of Proposition 12, and write, in this proof, $\underline{s} := V^*(\underline{v}, \bar{r}|0) = \underline{v} - m(\bar{r}|0)$, $\bar{s} := V^*(\bar{v}, \underline{r}|0) = \bar{v} - m(\underline{r}|0)$, $\check{s} := V^*(\underline{v}, r^\psi|0) = \underline{v} - m(r^\psi|0)$, so that $\underline{s} \leq \check{s} < \bar{s}$, $m(\cdot|0)$ being increasing, $\underline{r} < r^\psi \leq \bar{r}$ and $\underline{v} < \bar{v}$; $\underline{x} := \psi(\underline{r})$ and $\bar{x} := \psi(\bar{r})$;

and

$$A^*(x) := \int_{\underline{s}}^x F_V^*(t)^{N-1} dt, \quad A^P(x) := \int_{\underline{s} + \bar{x}}^x F_V^P(t)^{N-1} dt.$$

Step 1: The surplus change through the type. By Step 1 of the proof of Proposition 12, ψ is continuous with $\psi'(r) = (1 - \zeta)^2 \Delta_{\sigma^2} + \zeta(2 - \zeta) \sigma_0^2 \mathbf{I}\{r < r^*(\sigma_0^2)\}$ at $r \neq r^*(\sigma_0^2)$, so it is increasing on $[\underline{r}, r^\psi]$ and constant at $\bar{x}$ on $[r^\psi, \bar{r}]$, where $r^\psi = \bar{r}$ if $\Delta_{\sigma^2} > 0$ and $r^\psi = \min\{r^*(\sigma_0^2), \bar{r}\}$ if $\Delta_{\sigma^2} = 0$, as defined before the proposition, and $r^\psi > \underline{r}$ because $r^*(\sigma_0^2) > \underline{r}$, the assumption of section 3.2.3. Hence $\underline{x} < \bar{x}$, and ψ maps $[\underline{r}, r^\psi]$ onto $[\underline{x}, \bar{x}]$ with a continuous increasing inverse ψ^{-1} . Since $V(1, v_j, r_j|1) = V^*(v_j, r_j|0) + \psi(r_j) = v_j - c_1(r_j)$ at every type and c_1 is increasing, the bottom of the support of $V(1, v_j, r_j|1)$ is $V(1, \underline{v}, \bar{r}|1) = \underline{s} + \bar{x}$, and the definitions of u_{mh}^* and u_{mh}^P in section 3.2.3 read, at $s := V^*(v_i, r_i|0)$,

$$u_{\text{mh}}^*(v_i, r_i) = A^*(s), \quad u_{\text{mh}}^P(v_i, r_i) = A^P(s + \psi(r_i)), \\ \Delta u_{\text{mh}}^P(v_i, r_i) = A^P(s + \psi(r_i)) - A^*(s).$$

By Lemma 4 at $m(\cdot|0)$ and at c_1 , F_V^* is 0 at and below $\underline{s}$ and positive above it, and F_V^P is 0 at and below $\underline{s} + \bar{x}$ and positive above it; so A^* and A^P are continuous and non-decreasing, $A^*(x) = 0$ at $x \leq \underline{s}$, $A^P(x) = 0$ at $x \leq \underline{s} + \bar{x}$, each is increasing above its zero, and A^P maps $[\underline{s} + \bar{x}, \infty)$ onto $[0, \infty)$.

Step 2: The two extreme boosts. Since $V(1, v_j, r_j|1) = v_j - m(r_j|0) + \psi(r_j)$ and $\psi(r_j) \leq \bar{x}$, at every t ,

$$F_V^P(t + \bar{x}) - F_V^*(t) = \Pr(t < v_j - m(r_j|0) \leq t + \bar{x} - \psi(r_j)) \geq 0,$$

and the event requires $\psi(r_j) < \bar{x}$, that is $r_j < r^\psi$, and $v_j \in (t + m(r_j|0), t + c_1(r_j) + \bar{x}) \cap [\underline{v}, \bar{v}]$, since $m(\cdot|0) + \bar{x} - \psi = c_1 + \bar{x}$. At $t \leq \check{s}$ the event is empty: $r_j < r^\psi$ and c_1 increasing give $v_j \leq t + c_1(r_j) + \bar{x} < t + c_1(r^\psi) + \bar{x} = t + m(r^\psi|0) \leq \underline{v}$. At $t \in (\check{s}, \bar{s})$ it has positive probability: The types $r_j \in [\underline{r}, r^\psi]$ with $m(r_j|0) < \bar{v} - t$ form an interval $[\underline{r}, r_c)$ with $r_c > \underline{r}$, because $m(\underline{r}|0) < \bar{v} - t$, and either $r_c = r^\psi$ or $m(r_c|0) = \bar{v} - t$; in the first case $c_1(r_c) + \bar{x} = m(r^\psi|0) > \underline{v} - t$, in the second $c_1(r_c) + \bar{x} \geq m(r_c|0) = \bar{v} - t > \underline{v} - t$, $c_1 + \bar{x} - m(\cdot|0) = \bar{x} - \psi$ being nonnegative; so, c_1 being continuous, $c_1(r_j) + \bar{x} > \underline{v} - t$ on some $(r_b, r_c) \subseteq [\underline{r}, r_c)$, on which the valuation interval is non-degenerate and $f_{vr} > 0$. Hence, substituting $t = u - \bar{x}$ in A^P , at every $s \in [\underline{s}, \bar{s}]$,

$$A^P(s + \bar{x}) - A^*(s) = \int_{\underline{s}}^s [F_V^P(t + \bar{x})^{N-1} - F_V^*(t)^{N-1}] dt \geq 0,$$

with equality if and only if $s \leq \check{s}$, the integrand being nonnegative, 0 at $t \leq \check{s}$ and positive at $t \in (\check{s}, \bar{s})$, $x \mapsto x^{N-1}$ being increasing on $[0, 1]$. Likewise, since $\psi(r_j) \geq \underline{x}$,

$F_V^p(t + \underline{x}) = \Pr(v_j - m(r_j|0) \leq t + \underline{x} - \psi(r_j)) \leq F_V^*(t)$ at every t , and $F_V^p(t + \underline{x}) = 0 < F_V^*(t)$ at $t \in (\underline{s}, \underline{s} + \bar{x} - \underline{x})$, the bottoms of the two supports being $\underline{s} + \bar{x}$ and $\underline{s}$; hence, substituting $t = u - \underline{x}$ in A^p and extending its lower limit from $\underline{s} + \bar{x} - \underline{x}$ down to $\underline{s}$, where the integrand vanishes, at every $s > \underline{s}$,

$$A^p(s + \underline{x}) - A^*(s) = \int_{\underline{s}}^s \left[F_V^p(t + \underline{x})^{N-1} - F_V^*(t)^{N-1} \right] dt < 0.$$

Step 3: The threshold. Write, in this proof, for $s \in [\underline{s}, \check{s}]$,

$$\chi(s) := (A^p)^{-1}(A^*(s)) - s, \quad r^\dagger(s) := \psi^{-1}(\chi(s)),$$

$(A^p)^{-1}$ being the inverse of A^p on $[\underline{s} + \bar{x}, \infty)$, continuous and increasing from $[0, \infty)$ onto $[\underline{s} + \bar{x}, \infty)$ by Step 1. By Step 2 and the monotonicity of A^p ,

$$\chi(\underline{s}) = \bar{x}, \quad \underline{x} < \chi(s) \leq \bar{x} \quad \text{at } s > \underline{s}, \quad [\chi(s) = \bar{x} \iff s \leq \check{s}],$$

the first because $A^*(\underline{s}) = 0$, the second because $A^p(y) \leq A^p(s + \underline{x}) < A^*(s) = A^p(s + \chi(s))$ at every $y \leq s + \underline{x}$ while $A^*(s) \leq A^p(s + \bar{x})$, and the third because $A^*(s) = A^p(s + \bar{x})$ if and only if $s \leq \check{s}$. So χ is continuous, as is $r^\dagger(\cdot) = \psi^{-1} \circ \chi$, with values in $(\underline{r}, r^\psi]$, and $r^\dagger(s) = r^\psi$ if and only if $s \leq \check{s}$.

Since $A^*(s) = A^p(s + \chi(s))$, at every type with $V^*(v_i, r_i|0) = s$,

$$\Delta u_{\text{mh}}^p(v_i, r_i) = A^p(s + \psi(r_i)) - A^p(s + \chi(s)),$$

which at $s > \underline{s}$, where $A^*(s) > 0$ and so $s + \chi(s) > \underline{s} + \bar{x}$, has the sign of $\psi(r_i) - \chi(s)$, A^p being increasing on $[\underline{s} + \bar{x}, \infty)$ and 0 below; at $s = \underline{s}$ the only type is $(\underline{v}, \bar{r})$, with $\psi(\bar{r}) = \bar{x} = \chi(\underline{s})$ and $\Delta u_{\text{mh}}^p(\underline{v}, \bar{r}) = 0$. A type on a contour at or below the floor, $s \leq \check{s}$, has $r_i \geq r^\psi$, because $v_i - m(r_i|0) = s \leq \underline{v} - m(r^\psi|0)$ and $v_i \geq \underline{v}$ give $m(r_i|0) \geq m(r^\psi|0)$ and $m(\cdot|0)$ is increasing; so $\psi(r_i) = \bar{x} = \chi(s)$ and $\Delta u_{\text{mh}}^p(v_i, r_i) = 0$. At $s > \check{s}$, $\chi(s) < \bar{x}$ and $r^\dagger(s) < r^\psi$, and, ψ being increasing on $[\underline{r}, r^\psi]$ and equal to $\bar{x} > \chi(s)$ above it,

$$\begin{aligned} r_i < r^\dagger(s) &\iff \psi(r_i) < \chi(s), & r_i = r^\dagger(s) &\iff \psi(r_i) = \chi(s), \\ r_i > r^\dagger(s) &\iff \psi(r_i) > \chi(s). \end{aligned}$$

Combining the cases $s \leq \check{s}$ and $s > \check{s}$ with the sign of Δu_{mh}^p in $\psi(r_i) - \chi(s)$, at every type: $\Delta u_{\text{mh}}^p(v_i, r_i) < 0$ if and only if $r_i < r^\dagger(V^*(v_i, r_i|0))$; $\Delta u_{\text{mh}}^p(v_i, r_i) > 0$ if and only if $r_i > r^\dagger(V^*(v_i, r_i|0))$ and $V^*(v_i, r_i|0) > \check{s}$; and $\Delta u_{\text{mh}}^p(v_i, r_i) = 0$ if and only if $r_i = r^\dagger(V^*(v_i, r_i|0))$ or $V^*(v_i, r_i|0) \leq \check{s}$, which are parts (i)–(iii) with $\check{s} = V^*(\underline{v}, r^\psi|0)$. The unaffected set of part (iii) is the union of $\{V^*(v_i, r_i|0) \leq \check{s}\}$ and the contour $\{r_i = r^\dagger(V^*(v_i, r_i|0))\}$, the image of the graph of the continuous $r^\dagger(\cdot)$ under the Lipschitz map $(s, r) \mapsto (s + m(r|0), r)$

and so of Lebesgue measure zero, which gives the footnote to the statement. **Q.E.D.**

Proof of Proposition 15. Write, in this proof, $\underline{s} := V(0, \underline{v}, \bar{r}) = \underline{v} - \mu - \bar{r}\sigma^2$, F_V^{ban} for the CDF of $V(0, v_i, r_i)$, $\tilde{b}^{\text{ban}}$ for the ban's effective bid of Step 6 of the proof of Proposition 4, the $\tilde{b}$ of Lemma 6 at F_V^{ban} , $u^{\text{ban}}(v_i, r_i) := \int_{\underline{s}}^{V(0, v_i, r_i)} F_V^{\text{ban}}(t)^{N-1} dt$, and $M := \max_{j \neq i} \tilde{b}^{\text{ban}}(V(0, v_j, r_j))$.

The candidate. When every offer waives, the seller's payoff from accepting $(b_j, 0)$ is b_j at every c , so she accepts the highest price at every c , winning is uninformative about c , and a buyer who wins at price b earns $V(0, v_i, r_i) - b$: The stage game is the symmetric first-price auction in $V(0, v_i, r_i) = v_i - (\mu + r_i\sigma^2)$, whose cost side is continuous and increasing in r_i , so F_V^{ban} is continuous and increasing on $[\underline{s}, V(0, \bar{v}, \underline{r})]$ by Lemma 4. By Lemma 6(iv), in every symmetric monotone equilibrium in which every buyer waives, each buyer bids $\tilde{b}^{\text{ban}}(V(0, v_i, r_i))$, which by Lemma 6(iii) is continuous and increasing in $V(0, v_i, r_i)$ with $\tilde{b}^{\text{ban}}(\underline{s}) = \underline{s}$, so $M \geq \underline{s}$ and the highest $V(0, v_i, r_i)$ wins. A waiver $(x, 0)$ is accepted exactly on $\{M \leq x\}$, ties having probability zero, and $\tilde{b}^{\text{ban}}$ is the symmetric equilibrium bid of the auction in $V(0, v_i, r_i)$ (Lemma 6(v)), so $x = \tilde{b}^{\text{ban}}(V(0, v_i, r_i))$ maximises $\Pr(M \leq x)(V(0, v_i, r_i) - x)$ over x , with maximum $F_V^{\text{ban}}(V(0, v_i, r_i))^{N-1}[V(0, v_i, r_i) - \tilde{b}^{\text{ban}}(V(0, v_i, r_i))] = u^{\text{ban}}(v_i, r_i)$ by the definition of $\tilde{b}^{\text{ban}}$:

$$u^{\text{ban}}(v_i, r_i) = \max_x \Pr(M \leq x) (V(0, v_i, r_i) - x).$$

The seller accepts, the price being at least $\underline{s} > 0$ by Lemma 5. At $\kappa \geq \bar{\kappa}_{\text{as}}^* > \kappa^*$ (Corollary 15.1), $r^* > \bar{r}$ and $V^*(v_i, r_i) = V(0, v_i, r_i)$ at every type, so F_V^{ban} , $\tilde{b}^{\text{ban}}$ and u^{ban} are $F_{V, \text{as}}^*$, $\tilde{b}_{\text{as}}^*$ and u_{as}^* , and the bid is b^* of Proposition 1. It remains to price a deviation to a contingent offer $(b, 1)$, which the seller, knowing c , accepts if and only if $b - \kappa_S - \zeta c \geq M$.

Necessity. Let $\kappa < \bar{\kappa}_{\text{as}}^*$. The type $(\underline{v}, \bar{r})$ earns $u^{\text{ban}}(\underline{v}, \bar{r}) = 0$. For $\varepsilon > 0$, let him deviate to $(\underline{s} + \kappa_S + \varepsilon, 1)$, accepted on $\mathcal{E}_\varepsilon := \{M + \zeta c \leq \underline{s} + \varepsilon\}$. Since $\tilde{b}^{\text{ban}}$ is continuous with $\tilde{b}^{\text{ban}}(\underline{s}) = \underline{s}$, some $s_\varepsilon > \underline{s}$ has $\tilde{b}^{\text{ban}}(s_\varepsilon) \leq \underline{s} + \varepsilon/2$, and $\{V(0, v_j, r_j) \leq s_\varepsilon \text{ for every } j \neq i\} \cap \{\zeta c \leq \varepsilon/2\} \subseteq \mathcal{E}_\varepsilon$, so

$$\Pr(\mathcal{E}_\varepsilon) \geq F_V^{\text{ban}}(s_\varepsilon)^{N-1} F_c(\varepsilon/(2\zeta)) > 0,$$

c being independent of the types, F_V^{ban} increasing above $\underline{s}$, and $F_c(t) > 0$ for every $t > 0$. On $\mathcal{E}_\varepsilon$, $0 \leq \zeta c \leq \underline{s} + \varepsilon - M \leq \varepsilon$, so $\mathbb{E}[c \mid \mathcal{E}_\varepsilon] \leq \varepsilon/\zeta$ and $\text{Var}(c \mid \mathcal{E}_\varepsilon) \leq \mathbb{E}[c^2 \mid \mathcal{E}_\varepsilon] \leq \varepsilon^2/\zeta^2$, and his payoff conditional on acceptance is

$$\begin{aligned} & \underline{v} - (\underline{s} + \kappa_S + \varepsilon) - \kappa_B - (1 - \zeta)\mathbb{E}[c \mid \mathcal{E}_\varepsilon] - (1 - \zeta)^2 \bar{r} \text{Var}(c \mid \mathcal{E}_\varepsilon) \\ & \geq \bar{\kappa}_{\text{as}}^* - \kappa - \varepsilon - (1 - \zeta)\frac{\varepsilon}{\zeta} - (1 - \zeta)^2 \bar{r} \frac{\varepsilon^2}{\zeta^2}, \end{aligned}$$

by $\underline{v} - \underline{s} = \mu + \bar{r}\sigma^2 = \bar{\kappa}_{\text{as}}^*$ and $\kappa_B + \kappa_S = \kappa$. The right-hand side tends to $\bar{\kappa}_{\text{as}}^* - \kappa > 0$ as $\varepsilon \downarrow 0$; fix an ε at which it is positive. At every type (v_i, r_i) the deviation then pays at least $\Pr(\mathcal{E}_\varepsilon)$ times the right-hand side, a positive amount free of the type, because $v_i \geq \underline{v}$, $r_i \leq \bar{r}$ and $\mathcal{E}_\varepsilon$ does not depend on the type; u^{ban} being continuous with $u^{\text{ban}}(\underline{v}, \bar{r}) = 0$, it is smaller than that amount on a neighbourhood of $(\underline{v}, \bar{r})$, of positive probability, where the deviation is profitable: No symmetric monotone equilibrium has every buyer waiving.

Sufficiency. Let $\kappa \geq \bar{\kappa}_{\text{as}}^*$, and fix a type (v_i, r_i) and a deviation $(b, 1)$, accepted on $\{M \leq b - \kappa_S - \zeta c\} \subseteq \{M \leq b - \kappa_S\}$. Conditional on acceptance he earns $v_i - b - \kappa_B$ less his repair share and his risk penalty, both nonnegative, so his payoff from the deviation is at most

$$\Pr(M \leq b - \kappa_S) \max\{0, v_i - b - \kappa_B\} \leq \Pr(M \leq b - \kappa_S) \max\{0, V(0, v_i, r_i) - (b - \kappa_S)\} \\ \leq u^{\text{ban}}(v_i, r_i),$$

the first inequality because $(v_i - b - \kappa_B) - (V(0, v_i, r_i) - b + \kappa_S) = \mu + r_i\sigma^2 - \kappa \leq \bar{\kappa}_{\text{as}}^* - \kappa \leq 0$, and the second by the candidate's display at the waiver $x = b - \kappa_S$, or trivially where the maximum is 0. No contingent deviation pays, no waiver does, $\tilde{b}^{\text{ban}}$ being the equilibrium bid of the auction in $V(0, v_i, r_i)$, and accepting the highest price is optimal for the seller at every c : The candidate is an equilibrium. **Q.E.D.**

Proof of Corollary 15.1. By Proposition 1, $d^*(r) = \mathbf{I}\{r \geq r^*\}$, so nobody inspects if and only if $r^* \geq \bar{r}$, or equivalently, $\kappa \geq \zeta(2 - \zeta)\bar{r}\sigma^2 - \zeta^2 r_S \sigma^2 = \kappa^*$. Also,

$$\bar{\kappa}_{\text{as}}^* - \kappa^* = [\mu + \bar{r}\sigma^2] - [\zeta(2 - \zeta)\bar{r}\sigma^2 - \zeta^2 r_S \sigma^2] = \mu + (1 - \zeta)^2 \bar{r}\sigma^2 + \zeta^2 r_S \sigma^2 > 0.$$

Q.E.D.

Proof of Proposition 16. Step 1 gives the on-path play and disposes of contingent deviations, Step 2 prices a waiving deviation at the beliefs its acceptance induces, Step 3 compares the two, and Step 4 bounds the comparison by $\underline{\kappa}_{\text{as}}^*$ through the truncations of F_c .

Step 1: On-path play, and every contingent deviation. When every offer is contingent, the seller's payoff from accepting $(b_j, 1)$ at c is $b_j - \omega(c)$, with $\omega(c)$ common to all offers: She accepts the highest price at every c , winning is uninformative about c , and a buyer who wins at price b earns $v_i - b - \kappa_B - (1 - \zeta)\mu - (1 - \zeta)^2 r_i \sigma^2 = V(1, v_i, r_i) + \omega - b$. The stage game is the symmetric first-price auction in $V(1, v_i, r_i) + \omega$, of CDF $F_V^p(\cdot - \omega)$, continuous and increasing on $[V(1, \underline{v}, \bar{r}) + \omega, V(1, \bar{v}, \underline{r}) + \omega]$ by Lemma 4. Its bid of Lemma 6 is, by the substitution $t = t' + \omega$, $s + \omega - \int_{V(1, \underline{v}, \bar{r}) + \omega}^{s + \omega} F_V^p(t - \omega)^{N-1} dt / F_V^p(s)^{N-1} = \tilde{b}^p(s) + \omega$ at every $s \in (V(1, \underline{v}, \bar{r}), V(1, \bar{v}, \underline{r})]$, and $\tilde{b}^p(V(1, \underline{v}, \bar{r})) + \omega$ at the bottom by the extension of Lemma 6(iii). So each buyer bids $b^p(v_i, r_i) = \tilde{b}^p(V(1, v_i, r_i)) + \omega$ of

Proposition 2, continuous and increasing in $V(1, v_i, r_i)$ by Lemma 6(iii), the highest $V(1, v_i, r_i)$ wins, and by Lemma 6(iv) no other symmetric monotone equilibrium has every offer contingent. The seller accepts: The price is at least $V(1, \underline{v}, \bar{r}) + \omega$, so her payoff is at least $\underline{v} - \kappa - (1 - \zeta)\mu - \zeta c - (1 - \zeta)^2 \bar{r} \sigma^2 > 0$ by Lemma 5. Write, in this proof, $M := \max_{j \neq i} b^p(v_j, r_j)$ and G for its CDF. A contingent offer $(x, 1)$, on or off path, is accepted exactly on $\{M \leq x\}$, the offset cancelling, and earns $V(1, v_i, r_i) + \omega - x$; b^p being the symmetric equilibrium bid of this auction (Lemma 6(v)), $x = b^p(v_i, r_i)$ maximises that payoff over x , with maximum $F_V^p(V(1, v_i, r_i))^{N-1}[V(1, v_i, r_i) - \tilde{b}^p(V(1, v_i, r_i))] = u_{\text{as}}^p(v_i, r_i)$ by the definition of $\tilde{b}^p$:

$$u_{\text{as}}^p(v_i, r_i) = \max_x G(x) (V(1, v_i, r_i) + \omega - x).$$

No contingent deviation pays.

Step 2: A waiving deviation, at the beliefs its acceptance induces. Fix a type (v_i, r_i) and a deviation $(b, 0)$, and write $\tilde{b} := b + \kappa_S$. The seller's payoff from the waiver is b , so she accepts it if and only if $b \geq M - \omega(c)$, on $\mathcal{A} := \{M \leq \tilde{b} + \zeta c\}$. Write, in this proof, $h(c) := G(\tilde{b} + \zeta c)$, non-decreasing in c with values in $[0, 1]$, and $P := \mathbb{E}[h(c)]$, so that $\Pr(\mathcal{A}) = P$, the types being independent of c . If $P = 0$ the deviation never trades and earns $0 \leq u_{\text{as}}^p(v_i, r_i)$; let $P > 0$. Then $\mathbb{E}[c|\mathcal{A}] = \mathbb{E}[ch(c)]/P$ and $\text{Var}(c|\mathcal{A}) = \mathbb{E}[c^2h(c)]/P - \mathbb{E}[c|\mathcal{A}]^2$, and by the definition of π_B at these beliefs the deviation earns

$$P(v_i - b - \mathbb{E}[c|\mathcal{A}] - r_i \text{Var}(c|\mathcal{A})).$$

Step 3: The comparison, price by price. Step 1's display at $x = \tilde{b} + \zeta c$, averaged over c , gives $u_{\text{as}}^p(v_i, r_i) \geq \mathbb{E}[h(c)(V(1, v_i, r_i) + \omega - \tilde{b} - \zeta c)] = P(V(1, v_i, r_i) + \omega - \tilde{b} - \zeta \mathbb{E}[c|\mathcal{A}])$. Subtracting this from Step 2's display, with $\tilde{b} - b = \kappa_S$ and $v_i - V(1, v_i, r_i) - \omega = \kappa_B + (1 - \zeta)\mu + (1 - \zeta)^2 r_i \sigma^2$,

$$\begin{aligned} & P(v_i - b - \mathbb{E}[c|\mathcal{A}] - r_i \text{Var}(c|\mathcal{A})) - u_{\text{as}}^p(v_i, r_i) \\ & \leq P(\kappa + (1 - \zeta)\mu + (1 - \zeta)^2 r_i \sigma^2 - (1 - \zeta)\mathbb{E}[c|\mathcal{A}] - r_i \text{Var}(c|\mathcal{A})) \\ & = P\left(\kappa - \left[(1 - \zeta)(\mathbb{E}[c|\mathcal{A}] - \mu) + r_i(\text{Var}(c|\mathcal{A}) - (1 - \zeta)^2 \sigma^2)\right]\right). \end{aligned}$$

Step 4: The bracket through the truncations of F_c . Write, in this step, $B(t, r) := (1 - \zeta)(\mathbb{E}[c | c \geq t] - \mu) + r(\text{Var}(c | c \geq t) - (1 - \zeta)^2 \sigma^2)$ for $t \in [0, \bar{c})$, the minimand in the definition of $\underline{\kappa}_{\text{as}}^*$. On $[0, \bar{c})$, $1 - F_c(t) > 0$ and $|\mathbb{E}[c^j \mathbf{I}\{c \geq t\}] - \mathbb{E}[c^j \mathbf{I}\{c \geq t'\}]| \leq \bar{c}^j |F_c(t) - F_c(t')|$ for $j \in \{0, 1, 2\}$, so $B(\cdot, r)$ is continuous there, F_c being continuous; and $t \leq \mathbb{E}[c | c \geq t] \leq \bar{c}$ with $0 \leq \text{Var}(c | c \geq t) \leq (\bar{c} - t)^2/4$, so $B(t, r) \rightarrow (1 - \zeta)(\bar{c} - \mu) - r(1 - \zeta)^2 \sigma^2$ as $t \rightarrow \bar{c}$, the minimand's value at $t = \bar{c}$. Hence $B(\cdot, r)$ is continuous on $[0, \bar{c}]$, the minimum defining $\underline{\kappa}_{\text{as}}^*$ exists, and, $B(t, \cdot)$ being affine, $B(t, r_i) \geq \underline{\kappa}_{\text{as}}^*$ at every $t \in [0, \bar{c}]$ and every

$r_i \in [\underline{r}, \bar{r}]$. Since h is non-decreasing with values in $[0, 1]$, for every $x \in [0, h(\bar{c})]$ the set $\{c : h(c) > x\}$ is an interval with right endpoint $\bar{c}$ and left endpoint $t_x := \inf\{c : h(c) > x\}$, differing from $\{c \geq t_x\}$ by at most the point t_x , of probability zero, F_c being continuous. Hence, for every bounded ϕ , by Fubini's theorem,

$$\begin{aligned}\mathbb{E}[\phi(c)h(c)] &= \int_0^{h(\bar{c})} \mathbb{E}[\phi(c)\mathbf{I}\{h(c) > x\}] dx = \int_0^{h(\bar{c})} \mathbb{E}[\phi(c)\mathbf{I}\{c \geq t_x\}] dx \\ &= P \int_{[0, \bar{c})} \mathbb{E}[\phi(c) \mid c \geq t] d\nu(t),\end{aligned}$$

where ν is the image under $x \mapsto t_x$ of the measure $\Pr(c \geq t_x)dx/P$ on $\{x \in [0, h(\bar{c})] : t_x < \bar{c}\}$, a probability on $[0, \bar{c})$ by the display at $\phi \equiv 1$, the x with $t_x = \bar{c}$ contributing nothing to either side because $\Pr(c \geq \bar{c}) = 0$. At $\phi(c) = c$ and $\phi(c) = c^2$, by the law of total variance,

$$\begin{aligned}\mathbb{E}[c|\mathcal{A}] &= \int_{[0, \bar{c})} \mathbb{E}[c \mid c \geq t] d\nu(t), \\ \text{Var}(c|\mathcal{A}) &= \int_{[0, \bar{c})} \text{Var}(c \mid c \geq t) d\nu(t) + \int_{[0, \bar{c})} (\mathbb{E}[c \mid c \geq t] - \mathbb{E}[c|\mathcal{A}])^2 d\nu(t) \\ &\geq \int_{[0, \bar{c})} \text{Var}(c \mid c \geq t) d\nu(t),\end{aligned}$$

so, with $r_i \geq 0$, Step 3's bracket is at least $\int_{[0, \bar{c})} B(t, r_i) d\nu(t) \geq \underline{\kappa}_{\text{as}}^*$. Under $\kappa \leq \underline{\kappa}_{\text{as}}^*$, Step 3's right-hand side is nonpositive at every type and every price: No waiving deviation pays, and Step 1 covered the contingent ones. Finally, at $t = 0$ the truncated moments are μ and σ^2 , so $B(0, r) = \zeta(2-\zeta)r\sigma^2$ by $1-(1-\zeta)^2 = \zeta(2-\zeta)$, and $\underline{\kappa}_{\text{as}}^* \leq \zeta(2-\zeta)\underline{r}\sigma^2 < \mu + \bar{r}\sigma^2 = \bar{\kappa}_{\text{as}}^*$, as section 3.3 states. **Q.E.D.**

Proof of Proposition 17. Suppose the seller accepts the highest price at every c , so that a buyer's beliefs over c when his offer is accepted are the prior. The winner chooses d_i at a fixed price, so the election argument of the proof of Proposition 2 applies: Under assumption 2, $d_i = 1$ at every type, and a buyer who wins at price b_i earns $V(1, v_i, r_i) + \omega - b_i$. Every winner inspects, so the seller's payoff from accepting b_j at c is $b_j - \omega(c)$: She does accept the highest price at every c , which closes the supposition, and the stage game is the auction in $V(1, v_i, r_i) + \omega$ of Step 1 of the proof of Proposition 16, with the same bid, $\tilde{b}_{\text{as}}^{\text{p}}(V(1, v_i, r_i)) + \omega$ by the definitions of $\tilde{b}^{\text{p}}$ and $\tilde{b}_{\text{as}}^{\text{p}}$, the highest $V(1, v_i, r_i)$ winning and the seller never retaining. By Lemma 6(iv), every symmetric equilibrium in which the seller accepts the highest price at every c and whose bid is increasing has this bid, which is the footnote to the statement. **Q.E.D.**

Proof of Proposition 18. Under $\kappa \geq \bar{\kappa}_{\text{as}}^*$, $M_{\text{as}}^* = \max_i V(0, v_i, r_i)$, $F_{V_{\text{as}}}^*$ is the CDF of $V(0, v_i, r_i)$ and $\tilde{b}_{\text{as}}^*$ is the bid of Lemma 6 at it; $M_{\text{as}}^{\text{p}} = \max_i V(1, v_i, r_i)$, F_V^{p} is the CDF of $V(1, v_i, r_i)$ and $\tilde{b}_{\text{as}}^{\text{p}}$ is the bid at it. Step 1 compares the two effective valuations type by

type, Step 2 proves part (i), Step 3 part (ii) and the strictness claim of its footnote, and Step 4 the footnote's bounds in expectation.

Step 1: Type by type. At every type (v, r) and every $c \in [0, \bar{c}]$,

$$\begin{aligned} V(1, v, r) + \omega - \omega(c) - V(0, v, r) &= -\kappa + \zeta(2 - \zeta)r\sigma^2 + \zeta(\mu - c) \\ &\leq -\kappa + \zeta(2 - \zeta)\bar{r}\sigma^2 + \zeta(\mu - c) \\ &\leq -(1 - \zeta)\mu - (1 - \zeta)^2\bar{r}\sigma^2 - \zeta c \leq -(1 - \zeta)\mu - \zeta c < 0, \end{aligned}$$

the equality by the definitions of V , ω and $\omega(c)$ with $1 - (1 - \zeta)^2 = \zeta(2 - \zeta)$, the first inequality by $r \leq \bar{r}$, the second by $\kappa \geq \bar{\kappa}_{\text{as}}^* = \mu + \bar{r}\sigma^2$, the third by $\bar{r}\sigma^2 \geq 0$, and the last by $\mu > 0$, $\zeta < 1$ and $c \geq 0$.

Step 2: Part (i). Taking the maximum over i in Step 1, at every realization of the types and every c ,

$$\begin{aligned} \widetilde{M}_{\text{as}}^{\text{p}} &= \max_i V(1, v_i, r_i) + \omega - \omega(c) \leq M_{\text{as}}^* - \kappa + \zeta(2 - \zeta)\bar{r}\sigma^2 + \zeta(\mu - c) \\ &\leq M_{\text{as}}^* - (1 - \zeta)\mu - \zeta c < M_{\text{as}}^*, \end{aligned}$$

which is part (i) and the first sentence of the footnote.

Step 3: Part (ii). Write, in this proof, $s^*(q)$ and $s^{\text{p}}(q)$ for the quantiles of $V(0, v_i, r_i)$ and $V(1, v_i, r_i)$ at rank $q \in (0, 1]$, increasing by Lemma 4. By the proof of Lemma 1, $V(1, v, r) - V(0, v, r) = \Phi(r) \leq \Phi(\bar{r}) = -\kappa + \zeta(2 - \zeta)\bar{r}\sigma^2 - \zeta^2 r_{\text{S}}\sigma^2$ at every type, so $s^{\text{p}}(q) \leq s^*(q) + \Phi(\bar{r})$ at every q , the quantile of a pointwise smaller variable being smaller. By Lemma 6(i), at every $q \in (0, 1]$ and every c ,

$$\begin{aligned} \widetilde{b}_{\text{as}}^{\text{p}}(s^{\text{p}}(q)) + \omega - \omega(c) &= \int_0^q s^{\text{p}}(w) \frac{(N-1)w^{N-2}}{q^{N-1}} dw + \omega - \omega(c) \\ &\leq \int_0^q [s^*(w) + \Phi(\bar{r})] \frac{(N-1)w^{N-2}}{q^{N-1}} dw + \omega - \omega(c) \\ &= \widetilde{b}_{\text{as}}^*(s^*(q)) - \kappa + \zeta(2 - \zeta)\bar{r}\sigma^2 + \zeta(\mu - c) \\ &\leq \widetilde{b}_{\text{as}}^*(s^*(q)) - (1 - \zeta)\mu - \zeta c, \end{aligned}$$

the second equality by $\int_0^q (N-1)w^{N-2}/q^{N-1} dw = 1$ and $\Phi(\bar{r}) + \omega - \omega(c) = -\kappa + \zeta(2 - \zeta)\bar{r}\sigma^2 + \zeta(\mu - c)$, and the last inequality by Step 1. Let $q_{(N)}$ be the largest of N i.i.d. $\text{U}[0, 1]$ ranks, independent of c ; as in the proof of Lemma 6(ii), M_{as}^{p} and M_{as}^* have the laws of $s^{\text{p}}(q_{(N)})$ and $s^*(q_{(N)})$ and are independent of c , the types being independent of c , so the seller's surpluses $\widetilde{b}_{\text{as}}^{\text{p}}(M_{\text{as}}^{\text{p}}) + \omega - \omega(c)$ and $\widetilde{b}_{\text{as}}^*(M_{\text{as}}^*)$ have the laws of $Y^{\text{p}} := \widetilde{b}_{\text{as}}^{\text{p}}(s^{\text{p}}(q_{(N)})) + \omega - \omega(c)$ and $Y^* := \widetilde{b}_{\text{as}}^*(s^*(q_{(N)}))$. By the display at $q = q_{(N)}$, $Y^{\text{p}} \leq Y^* - (1 - \zeta)\mu$ at every $(q_{(N)}, c)$, so, at every x ,

$$\Pr(\widetilde{b}_{\text{as}}^{\text{p}}(M_{\text{as}}^{\text{p}}) + \omega - \omega(c) \leq x) = \Pr(Y^{\text{p}} \leq x) \geq \Pr(Y^* \leq x + (1 - \zeta)\mu)$$

$$\geq \Pr(Y^* \leq x) = \Pr(\tilde{b}_{\text{as}}^*(M_{\text{as}}^*) \leq x),$$

which is part (ii). For the footnote's strictness claim write $\underline{y} := V(1, \underline{v}, \bar{r}) + \omega - \omega(\bar{c})$ and $\bar{y} := \tilde{b}_{\text{as}}^*(V(0, \bar{v}, \underline{r}))$. By Lemma 6(iii), $q \mapsto \tilde{b}_{\text{as}}^p(s^p(q))$ and $q \mapsto \tilde{b}_{\text{as}}^*(s^*(q))$ are continuous and increasing on $(0, 1]$, with limits $V(1, \underline{v}, \bar{r})$ and $V(0, \underline{v}, \bar{r})$ as $q \rightarrow 0$; so $Y^p > \underline{y}$ and $V(0, \underline{v}, \bar{r}) < Y^* \leq \bar{y}$ almost surely, Y^* charges every non-degenerate subinterval of $(V(0, \underline{v}, \bar{r}), \bar{y})$, its preimage under the continuous increasing $q \mapsto \tilde{b}_{\text{as}}^*(s^*(q))$ being a non-degenerate interval of $(0, 1)$ and $q_{(N)}$ having density $Nq^{N-1} > 0$ there, and $\underline{y} < V(0, \underline{v}, \bar{r}) < \bar{y}$, the first by Step 1 at $(\underline{v}, \bar{r}, \bar{c})$. Hence, for some $\varepsilon > 0$ in the third row,

$$\Pr(Y^p \leq x) - \Pr(Y^* \leq x) \begin{cases} = 1 - 1 & \text{if } x \geq \bar{y}, \\ \geq \Pr(x < Y^* \leq x + (1 - \zeta)\mu) > 0 & \text{if } V(0, \underline{v}, \bar{r}) \leq x < \bar{y}, \\ \geq \Pr(q_{(N)} < \varepsilon, c > \bar{c} - \varepsilon) - 0 > 0 & \text{if } \underline{y} < x < V(0, \underline{v}, \bar{r}), \\ = 0 - 0 & \text{if } x \leq \underline{y}, \end{cases}$$

the first row because $Y^p \leq Y^* \leq \bar{y}$; the second by the display and Y^* charging $(x, \min\{x + (1 - \zeta)\mu, \bar{y}\})$; the third because $Y^* > V(0, \underline{v}, \bar{r}) > x$, while Y^p is continuous in $(q_{(N)}, c)$ with limit $\underline{y} < x$ as $q_{(N)} \rightarrow 0$ and $c \rightarrow \bar{c}$, so that $\{q_{(N)} < \varepsilon, c > \bar{c} - \varepsilon\} \subseteq \{Y^p \leq x\}$ for some $\varepsilon > 0$, an event of probability $\varepsilon^N(1 - F_c(\bar{c} - \varepsilon)) > 0$, $F_c(t) < 1$ for every $t < \bar{c}$; and the fourth because $Y^p > \underline{y}$ and $Y^* > \underline{y}$. Hence the inequality of part (ii) is strict if and only if $\underline{y} < x < \bar{y}$.

Step 4: In expectation. The mean of c is μ , so taking expectations in the first inequality of Step 2 gives $\Delta W_{\text{as}}^p \leq -\kappa + \zeta(2 - \zeta)\bar{r}\sigma^2$, and taking them, at $q = q_{(N)}$, in the inequality between the first and third rows of Step 3's first display, where $\tilde{b}_{\text{as}}^p(s^p(q_{(N)})) + \omega - \omega(c)$ and $\tilde{b}_{\text{as}}^*(s^*(q_{(N)}))$ have the laws of the seller's surplus in the two regimes and $\mathbb{E}[\zeta(\mu - c)] = 0$, gives $\Delta \Pi_{\text{as}}^p \leq -\kappa + \zeta(2 - \zeta)\bar{r}\sigma^2$; and $-\kappa + \zeta(2 - \zeta)\bar{r}\sigma^2 \leq -\mu - (1 - \zeta)^2\bar{r}\sigma^2 \leq -\mu$ by $\kappa \geq \mu + \bar{r}\sigma^2$. **Q.E.D.**